مسابقة في مادة الفيزياء	الاسم:
المدة: ساعة ونصف	الرقم:

This exam is formed of three obligatory exercises in three pages.
The use of non-programmable calculator is recommended.

Exercise 1 (7 pts)

Motion of a block on an inclined plane

A block (M), considered as a particle of mass m = 0.5 kg, can move on a straight track AF, situated in a vertical plane and inclined by an angle $\alpha = 30^o$ with the horizontal. Point A is taken as an origin of the x-axis confounded with (AF). A massless spring of natural length $\ell_0 = 50$ cm and force constant k is placed on the inclined track; one of its ends is fixed at F, and the other end is free at O (Doc. 1).

(M) is released without initial velocity from the top A of the track AF, and then it passes through point B with a velocity $\vec{V}_B$ of magnitude $V_B = \sqrt{2}$ m/s and through point C with a velocity $\vec{V}_C$ of magnitude $V_C = \sqrt{2.4}$ m/s. After point C, (M) continues its motion without friction, hits the spring, and compresses it by a distance OO'.

The aim of this exercise is to determine k.

Take:

- the horizontal plane containing (FH) as a reference level for the gravitational potential energy;
- AB = BC = 20 cm, AF = 1.6 m;
- $g = 10$ m/s^2.

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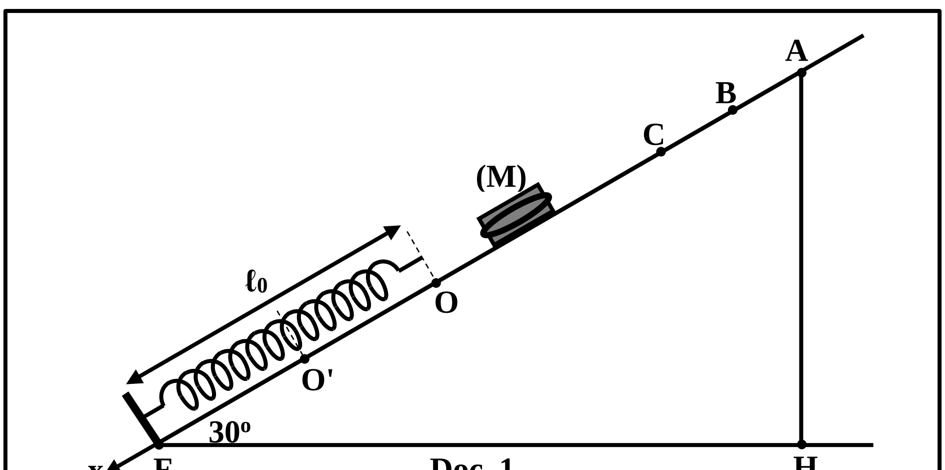


Doc. 1

1) Calculate the mechanical energy of the system [(M), Earth] at A, B and C.
2) Deduce that:
   2.1) the motion of (M) between A and B takes place without friction;
   2.2) (M) is submitted to a force of friction $\vec{f}$ during its motion between B and C.
3) Show that the internal energy of the system [(M), Earth, Track, Atmosphere] increases by 0.4 J during the motion of (M) between B and C.
4) Determine the magnitude f of the force of friction $\vec{f}$, supposed constant and parallel to the displacement, exerted on (M) during its motion between B and C.
5) The mechanical energy of the system [(M), Earth] at O is $ME_O = 3.6$ J. Why?
6) (M) reaches O, and compresses the spring by a maximum distance OO' = 24 cm.
   Determine the value of k.

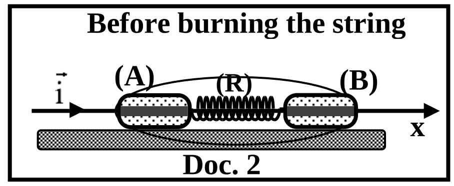
Exercise 2 (6 pts)

Launching of two pucks

An experimental device is made up of:

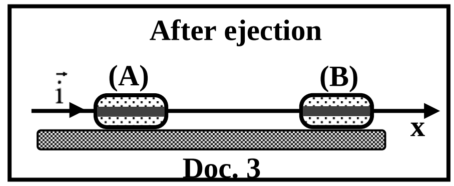
- two pucks (A) and (B), of respective masses m_A and m_B, able to move without friction on a horizontal rail;
- a spring (R) of negligible mass and of stiffness k = 100 N/m.

(R) is compressed between (A) and (B) by means of a light string to form a system at rest (Doc. 2).



Doc. 2

We burn the string, (A) and (B) are then ejected. Just after ejection, (A) and (B) move on the horizontal rail and their centers of mass move along the horizontal x-axis of unit vector $\vec{i}$ with the velocities $\vec{v}_A$ and $\vec{v}_B$ respectively (Doc. 3).



Doc. 3

The aim of this exercise is to study the effect of the mass of a puck on its speed after ejection.

Take the horizontal plane containing the x-axis as a reference level for gravitational potential energy.

1) Before burning the string, the system [(A), (B), (R)] possesses a certain form of energy « E ».

1.1) In what form is this energy stored?

1.2) Calculate the value of « E » knowing that the spring is compressed by 4 cm.

2) After ejection, the linear momentum of the system [(A), (B)] is conserved. Justify.

3) Deduce that (A) and (B) are ejected in opposite directions.

4) The used experimental device permits to measure the speed v_A of (A) after ejection. Two experiments are carried out with this device.

4.1) **Experiment 1:** The two pucks have the same mass $m_A = m_B$ = 500 g. Just after ejection, (A) moves with a velocity $\vec{v}_A = -0.4\ \vec{i}$ (v_A in m/s).

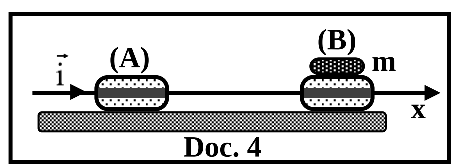
4.1.1) Determine $\vec{v}_B$.

4.1.2) Deduce the value of the kinetic energy KE of the system [(A), (B)] just after ejection.

4.1.3) Compare the obtained values of KE and « E ». Conclude.

4.2) **Experiment 2:** We repeat experiment 1 by adding an object of mass m = 100 g to puck (B), the mass of (A) remains the same (Doc. 4). Just after ejection, (A) moves with a velocity $\vec{v}_A = -0.42\ \vec{i}$ (v_A in m/s).

Determine $\vec{v}_B$.



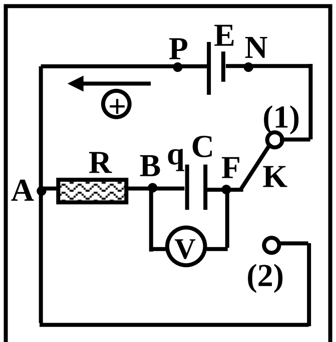
Doc. 4

5) Deduce the effect of the increase in mass of a puck on its speed after ejection.

Exercise 3 (7 pts)

Time constant of RC series circuit

The aim of this exercise is to determine the time constant τ of RC series circuit during charging and discharging of a capacitor and the capacitance of a capacitor.



Doc.5

For this purpose, we set up the circuit of document 5 that includes:

- a capacitor, initially uncharged, of capacitance C;
- a resistor of resistance R = 100 kΩ;
- an ideal battery of constant voltage $U_{PN} = E = 12$ V;
- a voltmeter (V) connected in parallel across the terminals of the capacitor;
- a double switch K.

1) Charging the capacitor

At the instant $t_0 = 0$, K is placed in position (1) and the charging process of the capacitor starts.

1.1) Show that the differential equation that describes the variation of the voltage $u_{BF} = u_C$ across the capacitor is: $RC\frac{du_C}{dt} + u_C = E$.

1.2) The solution of this differential equation has the form: $u_C = E - E\,e^{\frac{-t}{\tau}}$ where τ is constant. Determine the expression of τ in terms of R and C.

1.3) At $t_1 = 7$ s, the voltage across the terminals of the capacitor equals $\frac{E}{2}$. Determine the value of τ.

1.4) Deduce the value of C.

2) Discharging the capacitor

The capacitor is completely charged.
At an instant $t_0 = 0$, taken as an initial time, the switch K is turned to position (2); the phenomenon of discharging of the capacitor thus starts.

The variation of u_C during this case is given by: $u_C = E\ e^{\frac{-t}{\tau}}$.

2.1) Show that: $\ell n\left(\frac{E}{u_C}\right) = \frac{1}{\tau} \times t$.

2.2) The table below, gives different values of $u_{BF} = u_C$ measured by the voltmeter (V) at different instants of time t .

t (s)	0	5	10	20	30	40	50
u_C (V)	12	7.3	4.4	1.6	0.6	0.2	0.08
$\ell n\left(\frac{E}{u_C}\right)$							

2.2.1) Copy and complete the table.

2.2.2) Trace, on a graph paper, the curve that represents $\ell n\left(\frac{E}{u_C}\right)$ as a function of time.

Take the scale:

- on the abscissa axis 1 cm ↔ 5 s;
- on the ordinate axis 1 cm ↔ 1.

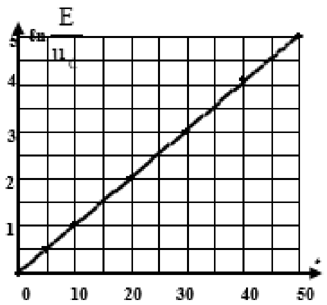
2.2.3) Referring to the obtained curve, show that: $\ell n\left(\frac{E}{u_C}\right) = 0.1 \times t$ (S.I).

2.3) Deduce the values of the time constant τ of the circuit and the capacitance C of the capacitor.

مسابقة في مادة الفيزياء
أسس التصحيح - إنكليزي

Exercise 1 (7 pts)	Motion of a block on an inclined plane	
Part	**Answer**	**Note**
1	$h_A = AF\sin30^o = 1.6 \times 0.5 = 0.8$ m $ME_A = mgh_A + 0 = 0.5 \times 10 \times 0.8 = 4$ J $ME_B = mgh_B + \frac{1}{2}m\,V_B^2 = 0.5 \times 10 \times 1.4 \times \sin30^o + \frac{1}{2} \times 0.5 \times (\sqrt{2})^2 = 4$ J $ME_C = mgh_C + \frac{1}{2}m\,V_C^2 = 0.5 \times 10 \times 1.2 \times \sin30^o + \frac{1}{2} \times 0.5 \times (\sqrt{2.4})^2 = 3.6$ J	0.5 0.5 1 0.5
2.1	Since $ME_A = ME_B = 4$ J	0.25
2.2	Since $ME_C < ME_B$; so, ME decreases thus friction exists between B and C	0.25
3	The system [(M), Earth, Track, Atmosphere] is energy isolated. So its total energy E is conserved, E = ME + U = constant So, $\Delta U = -\Delta(ME) = -(3.6 - 4) = 0.4$ J $\Delta U > 0$ so it's internal energy increases by 0.4 J.	1
4	The variation in mechanical energy equals the work of friction: $\Delta ME = W_{\vec{f}}$ so $\Delta(ME) = -0.4 = -f \times BC = -f \times 0.2$. So, f = 2 N	1
5	Since $ME_O = ME_C = 3.6$ J	0.25
6	$ME_O = ME_{O'} = KE_{O'} + PE_{g(O')} + PE_{el(O')} = 0 + mgh_{O'} + \frac{1}{2}kx^2$ But x = 0.24 m and v = 0, (maximum compression) $\sin30^o = \frac{h_{O'}}{FO'}$ with FO' = FO – OO' = 50 – 24 = 26 cm = 0.26 m So $h_{O'} = 0.26 \times \sin30^o = 0.13$ m, Thus, $3.6 = 0.5 \times 10 \times 0.13 + \frac{1}{2} \times k \times 0.24^2$ Hence, k = 102.43 N/m	1.75

Exercise 2 (6 pts) Launching of two pucks		
Part	**Answer**	**Note**
1.1	In the form of elastic potential energy in the spring.	**0.5**
1.2	$E = \frac{1}{2}k\,x^2 = \frac{1}{2}\times 100\times(0.04)^2 = 0.08$ J	**0.75**
2	The external forces on the system [(A), (B)] are: Weight $\overrightarrow{W_A}$ and the normal reaction $\overrightarrow{N_A}$: $\overrightarrow{W_A} + \overrightarrow{N_A} = \vec{0}$ Weight $\overrightarrow{W_B}$ and the normal reaction $\overrightarrow{N_B}$: $\overrightarrow{W_B} + \overrightarrow{N_B} = \vec{0}$ So, the sum of external forces acting on the system [(A), (B)] is nil. By Newton's second law, $\sum \overrightarrow{F_{ext}} = \frac{d\vec{P}}{dt}$, Thus, linear momentum of the system [(A), (B)] is conserved.	**0.75**
3	$\vec{P}_{before} = \vec{P}_{after}$ so $\vec{0} = m_A\vec{v}_A + m_B\vec{v}_B$ thus $\vec{v}_A = -\frac{m_B}{m_A}\vec{v}_B$ Thus, $\vec{v}_A$ has a direction opposite to $\vec{v}_B$.	**1**
4.1.1	$\vec{v}_B = -\frac{m_A}{m_B}\vec{v}_A$, so $\vec{v}_B = +0.4\,\vec{i}$ (v_B in m/s)	**0.25**
4.1.2	$KE = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}\times 0.5\times 0.16 + \frac{1}{2}\times 0.5\times 0.16 = 0.08$ J	**1**
4.1.3	KE = E = 0.08 J Conclusion : All stored elastic potential energy is converted into kinetic energy.	**0.5** **0.5**
4.2	$\vec{v}_B = -\frac{m_A}{m_B}\vec{v}_A$, so $\vec{v}_B = -\frac{0.5}{0.6}\times(-0.42\,\vec{i}) = 0.35\,\vec{i}$ m/s	**0.25**
5	As the mass of the puck increases, its speed decreases.	**0.5**

Exercise 3 (7 pts)	**Time constant of RC series circuit**	
Part	**Answer**	**Note**
1.1	Law of addition of voltages: $u_{PN} = u_{PA} + u_{AB} + u_{BF} + u_{FN}$ $E = R\,i + u_C$, but $i = \frac{dq}{dt}$ and $q = C\,u_C$, so $i = C\frac{du_C}{dt}$ Then: $E = R\,C\frac{du_C}{dt} + u_C$	**1**
1.2	$u_C = E - E\,e^{\frac{-t}{\tau}}$ so $\frac{du_C}{dt} = \frac{E}{\tau}\,e^{\frac{-t}{\tau}}$ Replace u_C and $\frac{du_C}{dt}$ in the differential equation: $R\,C\,[\frac{E}{\tau}\,e^{\frac{-t}{\tau}}] + E - E\,e^{\frac{-t}{\tau}} = E$ then: $E\,e^{\frac{-t}{\tau}}[\frac{RC}{\tau} - 1] + E = E$ So $E\,e^{\frac{-t}{\tau}}[\frac{RC}{\tau} - 1] = 0$ but $E\,e^{\frac{-t}{\tau}} \neq 0$ thus $\frac{RC}{\tau} - 1 = 0$; therefore, $\tau = R\,C$	**0.75**
1.3	$u_C = E - E\,e^{\frac{-t}{\tau}}$ so $\frac{E}{2} = E - E\,e^{\frac{-t_1}{\tau}}$, we get $E\,e^{\frac{-t_1}{\tau}} = \frac{E}{2}$ Then, $e^{\frac{-t_1}{\tau}} = \frac{1}{2}$; so $\frac{-t_1}{\tau} = -\ell n2$, then $\tau = \frac{t_1}{\ell n\,2} = \frac{7}{0.693} = 10.10\text{ s} \approx 10\text{ s}$	**0.75**
1.4	$\tau = R\,C$, so $10.10 = 10^5$, then $C \approx 10^{-4}$ F	**0.5**
2.1	$u_C = E\,e^{\frac{-t}{\tau}}$ so $\frac{E}{u_C} = \frac{1}{e^{\frac{-t}{\tau}}} = e^{\frac{t}{\tau}}$. Then, $\ell n\frac{E}{u_C} = \ell n\;e^{\frac{t}{\tau}}$. So, $\ell n\frac{E}{u_C} = \frac{t}{\tau}$ Hence, $\ell n\frac{E}{u_C} = \frac{1}{\tau}\times t$	**0.5**
2.2.1	$\ell n\frac{E}{u_C}$ | 0 | 0.5 | 1 | 2 | 3 | 4.1 | 5	**1**
2.2.2		**1**
2.2.3	The graph is a straight line passing through the origin whose equation is: $\ln\frac{E}{u_C} = \text{slope}\times t$; slope = (5-0)/(50-0) = 0.1 Thus, $\ln\frac{E}{u_C} = 0.1\times t$	**0.5**
2.3	Slope = 0.1 = $\frac{1}{\tau}$ which gives $\tau = 10$ s But, $\tau = RC$; thus, $C = \tau/R = 10/10^5 = 10^{-4}$ F	**0.5** **0.5**