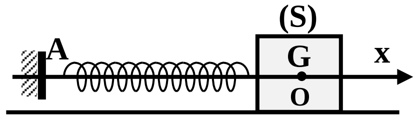
This exam is formed of three obligatory exercises in three pages.
The use of non-programmable calculator is recommended.

Exercise 1 (7 pts)

Mechanical oscillations

A mechanical oscillator consists of a block (S) of mass m and a spring of negligible mass and force constant k = 20 N/m.
The spring is connected from one of its ends to a fixed support A. (S) is attached to the other end of the spring and it may slide without friction on a horizontal support (Doc. 1).



Doc. 1

At equilibrium, G, the center of mass of (S), coincides with the origin O of the x-axis.
At the instant $t_0 = 0$, G is at O and we launch (S) with a velocity $\vec{v}_0 = v_0 \vec{i}$; thus, (S) undergoes mechanical oscillations with an amplitude X_m.

At an instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic value of its velocity is $v = x' = \frac{dx}{dt}$.

The aim of this exercise is to study for this oscillator the effect of v_0 on the oscillation amplitude X_m.
Take:

- the horizontal plane passing through G as a reference level for gravitational potential energy;
- $g = 10\ m/s^2$ and $\pi^2 = 10$.

1) Theoretical study

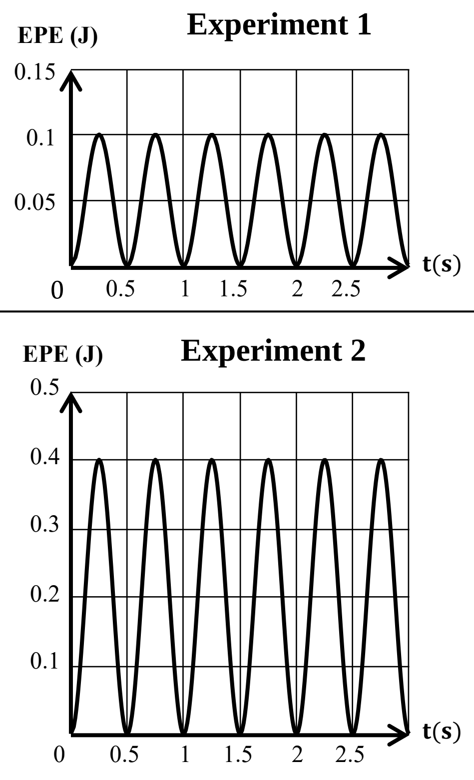
1.1) Write the expression of the mechanical energy ME of the system (Oscillator , Earth) in terms of x, m, k and v.

1.2) Determine the second order differential equation that governs the variation of x.

1.3) Deduce the expression of the proper (natural) period T_0 of the oscillations in terms of m and k.

2) Experimental study

An appropriate device gives the elastic potential energy EPE of the oscillator as a function of time for two different experiments, experiment 1 and experiment 2 (Doc. 2).



Doc. 2

2.1) Use the graphs of document 2 in order to:

2.1.1) justify that the oscillations of (S) are undamped.

2.1.2) copy and then complete the following table:

	Experiment 1	Experiment 2
The maximum value of EPE		
The value of the period T_E of EPE		

2.2) Show that m = 0.5 kg knowing that $T_0 = 2T_E$.

2.3) Show that $X_{m(2)} = 2\ X_{m(1)}$, where $X_{m(1)}$ and $X_{m(2)}$ are the amplitudes of the oscillations in experiments 1 and 2 respectively.

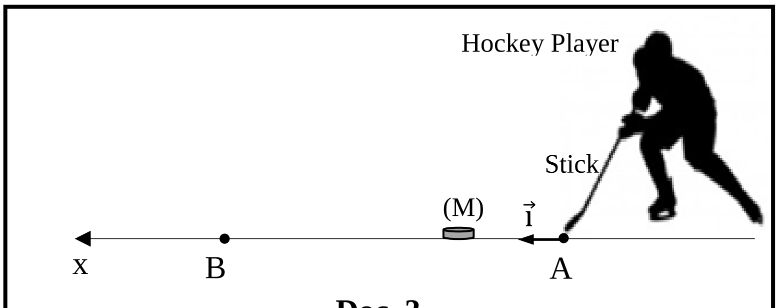
2.4) Determine the values of v_0 for the two experiments.

2.5) Deduce whether X_m increases, decreases, or remains the same as v_0 increases.

Exercise 2 (6.5 pts)

Motion of a hockey puck

The purpose of this exercise is to study the motion of a hockey puck (M).
(M), taken as a particle of mass m = 170 g, can slide on a horizontal ice rink. A hockey player hits puck (M) with his stick from point A (Doc. 3).
Take the horizontal plane passing through (M) as a reference level for gravitational potential energy.



Doc. 3

1) The collision between (M) and the stick occurs in a very short time. Choose the correct sentence out of the three following sentences.
Sentence 1: During this collision, the linear momentum and the kinetic energy of the system [Stick , (M)] are necessarily conserved.
Sentence 2: During this collision, the linear momentum of the system [Stick , (M)] is conserved but the kinetic energy of this system is not necessarily conserved.
Sentence 3: During this collision, the linear momentum of the system [Stick , (M)] is not necessarily conserved but the kinetic energy of this system is necessarily conserved.

2) Just after the collision, (M) is launched from point A with a velocity $\vec{v}_A = 18\,\vec{i}$ (m/s). Puck (M) moves on the ice rink along an x-axis, and it stops at point B after travelling a distance AB = 54 m during a time Δt (Doc. 3).

2.1) Calculate the mechanical energy of the system [(M) , Earth] at A and then at B.

2.2) Deduce that (M) is submitted to a friction force $\vec{f}$ during its motion between A and B.

2.3) Given that the value f of $\vec{f}$ is constant. Deduce that f = 0.51 N.

2.4) Name the external forces acting on (M) between A and B, and then draw, not to scale, a diagram for these forces.

2.5) Show that the sum of these forces is $\sum \vec{F}_{ext} = -\,0.51\,\vec{i}$ (N).

2.6) Determine the linear momenta of (M), « $\vec{P}_A$ » at point A and « $\vec{P}_B$ » at point B.

2.7) Deduce the variation $\Delta\vec{P}$ of the linear momentum of (M) during Δt.

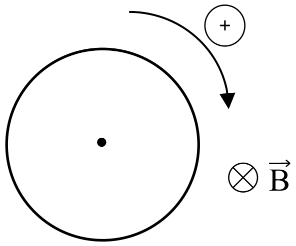
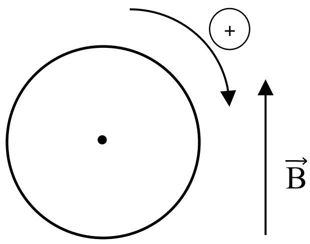
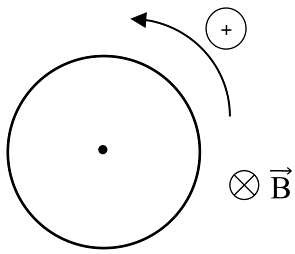
2.8) Calculate Δt knowing that $\Delta\vec{P} = (\sum \vec{F}_{ext})\,\Delta t$.

Exercise 3 (6.5 pts)

Electromagnetic induction

The purpose of this exercise is to determine the direction of the induced current in a circular loop by two different methods.
Consider a circular conducting loop of radius r = 10 cm and resistance R = 2 Ω. The loop is placed in a uniform magnetic field $\vec{B}$.

1) Document 4 shows three different cases.

1st case	2nd case	3rd case
The plane of the loop is perpendicular to the magnetic field lines of $\vec{B}$.	The plane of the loop is parallel to the magnetic field lines of $\vec{B}$.	The plane of the loop is perpendicular to the magnetic field lines of $\vec{B}$.
		
Doc. 4		

Match each of the following sentences 1, 2 and 3 to its appropriate case. Justify.

Sentence 1: The magnetic flux through the loop is zero.
Sentence 2: The magnetic flux through the loop is positive.
Sentence 3: The magnetic flux through the loop is negative.

2) Consider the first case of document 4. During the time interval [0 , 2 s], the value B of the magnetic field $\vec{B}$ decreases with time according to the relation:

$$B = -0.04\,t + 0.8 \quad (SI)$$

2.1) A current is induced in the loop during the time interval [0 , 2s]. Justify.
2.2) Apply Lenz's law in order to specify the direction of the induced current.
2.3) Determine the expression of the magnetic flux crossing the loop as a function of time.
2.4) Deduce the value of the induced electromotive force « e ».
2.5) The current carried by the loop is given by the relation $i = \frac{e}{R}$. Deduce the value and the direction of « i ».
2.6) Compare the direction of the induced current obtained in part (2.5) to that obtained in part (2.2).

وزارة التربية والتعليم العالي
المديرية العامّة للتربية
دائرة الامتحانات الرسميّة

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

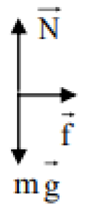
دورة العام 2022 الإستثنائية

مسابقة في مادة الفيزياء
المدة: ساعة ونصف

الاسم:
الرقم:

Exercise 1 : Mechanical oscillations (7 pts)

Part		Answer	Mark
1	1.1	$ME = KE + EPE = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$	0.5
	1.2	Friction is neglected, then the mechanical energy is conserved. Or: The sum of the works done by the nonconservative forces is zero, then ME is conserved. Then, $\frac{dME}{dt} = 0$, so $m v v' + k x x' = 0$ { $v = x'$ and $v' = x''$ } $v (m x'' + k x) = 0$, but $v = 0$ is rejected , so $m x'' + k x = 0$; therefore, $x'' + \frac{k}{m} x = 0$	1
	1.3	The differential equation is of the form: $x'' + \omega_0^2 x = 0$ with $\omega_0 = \sqrt{\frac{k}{m}}$ $T_0 = \frac{2\pi}{\omega_0}$, then $T_0 = 2\pi\sqrt{\frac{m}{k}}$	1
2	2.1.1	$EPE_{max} = \frac{1}{2} k X_m^2$ = constant. k is constant, then X_m is constant; therefore, the oscillations are undamped.	0.5
	2.1.2	| | Experiment 1 | Experiment 2 | | The maximum value of EPE | 0.1 J | 0.4 J | | The value of the period T_E of EPE | 0.5 s | 0.5 s |	0.5 0.5
	2.2	$T_0 = 2 T_E = 2 (0.5) = 1$ s $T_0 = 2\pi\sqrt{\frac{m}{k}}$, then $T_0^2 = 4\pi^2 \frac{m}{K}$, so $m = \frac{k T_0^2}{4\pi^2}$ $m = \frac{20 \times 1}{4 \times 10}$, hence $m = 0.5$ kg	0.5
	2.3	Experiment 1 : $EPE_{max} = 0.1 = \frac{1}{2} k X_{m(1)}^2$ … eq(1) Experiment 2 : $EPE_{max} = 0.4 = \frac{1}{2} k X_{m(2)}^2$ …eq(2) ; Dividing eq(2) by eq(1) gives: $\frac{0.4}{0.1} = \frac{X_{m(2)}^2}{X_{m(1)}^2}$, then $4 = (\frac{X_{m(2)}}{X_{m(1)}})^2$, hence $2 = \frac{X_{m(2)}}{X_{m(1)}}$ Therefore, $X_{m(2)} = 2 X_{m(1)}$	0.5
	2.4	ME = constant , then $ME = EPE_{max} = KE_{max}$, so $EPE_{max} = \frac{1}{2} m v_0^2$ Experiment 1 : $0.1 = \frac{1}{2} (0.5) v_{0(1)}^2$, then $v_{0(1)} = 0.63$ m/s Experiment 2 : $0.4 = \frac{1}{2} (0.5) v_{0(2)}^2$, then $v_{0(2)} = 1.26$ m/s	0.5 0.25 0.25
	2.5	v_o in experiment 2 is greater than v_o in experiment 1 ($v_{0(2)} > v_{0(1)}$) and $X_{m(2)} > 2 X_{m(1)}$; therefore, as v_0 increases X_m increases.	0.5 0.5

Exercise 2: Motion of a hockey puck (6.5 pts)			
Part		**Answer**	**Mark**
1		Sentence 2	**0.5**
2	**2.1**	$GPE_A = GPE_B = 0$ since (M) is at the reference level. $ME_A = KE_A + GPE_A = \frac{1}{2} m v_A^2 + 0 = \frac{1}{2} \times 0.17 \times 18^2$, then $ME_A = 27.54$ J $KE_B = 0$ since (M) stops at point B. $ME_B = KE_B + GPE_B = 0 + 0$, then $ME_B = 0$	**0.75** **0.25**
	2.2	$ME_B < ME_A$, then (M) is submitted to a friction force.	**0.25**
	2.3	$\Delta ME = W_{\vec{f}} = \vec{f}.\overrightarrow{AB}$, then $ME_B - ME_A = - f \times AB$ $0 - 27.54 = - f \times 54$, hence $f = 0.51$ N	**1**
	2.4	Forces acting on (M) : The weight $m\vec{g}$ The normal force $\vec{N}$ exerted by the ice rink The friction force $\vec{f}$  	**0.5** **0.5**
	2.5	$\sum \vec{F}_{ext} = m\vec{g} + \vec{N} + \vec{f}$, but $m\vec{g} + \vec{N} = \vec{0}$ Then, $\sum \vec{F}_{ext} = \vec{f} = -f\vec{i} = - 0.51\,\vec{i}$ (N)	**0.75**
	2.6	$\vec{P}_A = m\,\vec{v}_A = 0.17 \times 18\,\vec{i}$, then $\vec{P}_A = 3.06\,\vec{i}$ (kg.m/s) $\vec{P}_B = m\,\vec{v}_B = m\,(\vec{0})$, then $\vec{P}_B = \vec{0}$	**0.75** **0.25**
	2.7	$\Delta\vec{P} = \vec{P}_B - \vec{P}_A = \vec{0} - 3.06\,\vec{i}$, then $\Delta\vec{P} = - 3.06\,\vec{i}$ (kg.m/s)	**0.5**
	2.8	$\Delta t = \frac{\Delta\vec{P}}{\sum \vec{F}_{ext}} = \frac{-3.06\,\vec{i}}{-0.51\,\vec{i}}$, then $\Delta t = 6$ s	**0.5**

Exercise 3 (6.5 pts)	Electromagnetic induction	
Part	**Answer**	**Mark**
1	Sentence 1 corresponds to the 2nd case, because: • $\phi = \vec{B} \cdot \vec{n}\, S = B\, S \cos(\vec{B}, \vec{n}) = B\, S \cos 90^o = 0$ • **or** the plane of the loop is parallel to the field lines • **or** the field lines do not cross the loop Sentence 2 corresponds to the 1nd case, because: • the angle between the unit vector $\vec{n}$ and $\vec{B}$ is zero • **or** $\phi = B\, S \cos 0^o = B\, S\, (1)$, but B and S are positive ; therefore, ϕ is positive. Sentence 3 corresponds to the 3rd case, because: • the angle between the unit vector $\vec{n}$ and $\vec{B}$ is 180° • **or** $\phi = B\, S \cos 180^o = -\, B\, S$, but B and S are positive ; therefore, ϕ is negative.	**0.5** **0.5** **0.5**
2.1	During [0, 2s], the magnitude B of $\vec{B}$ changes, then the loop is crossed by a variable magnetic flux; therefore, the loop becomes the seat of induced emf. The loop forms a closed circuit, then it carries electric current.	**0.75**
2.2	During [0, 2s], B decreases, then the direction of the induced magnetic field is the same as that of $\vec{B}$ in order to oppose the decrease in B. According to the right hand rule, the induced current passes in the loop in the chosen positive sense (clockwise).	**0.75**
2.3	$\phi = \vec{B} \cdot \vec{n}\, S = B\, S \cos(\vec{B}, \vec{n}) = B\, S \cos 0^o = B\, S = B\, \pi\, r^2$ $\phi = (-\, 0.04\, t + 0.8) \times \pi \times (0.1)^2$ $\phi = -\, 4\pi \times 10^{-4}\, t + 8\pi \times 10^{-4}$ (SI)	**1**
2.4	$e = -\frac{d\varphi}{dt} = -\,(-\, 4\pi \times 10^{-4})$, then $e = 4\pi \times 10^{-4}$ V	**1**
2.5	$i = \frac{e}{R} = \frac{4\pi \times 10^{-4}}{2} = 6.3 \times 10^{-3}$A i > 0, then the current is in the chosen positive sense (Clockwise).	**1**
2.6	The direction is the same in the two parts.	**0.5**