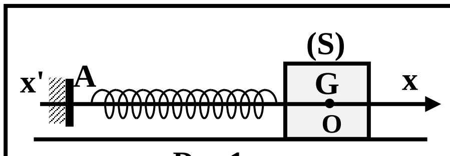
مسابقة في مادة الفيزياء	الاسم:
المدة: ساعة ونصف	الرقم:

This exam is formed of three obligatory exercises in three pages.
The use of non-programmable calculator is recommended.

Exercise 1 (7 pts)

Mechanical oscillations

A mechanical oscillator consists of a block (S) of mass m = 50 g and a massless spring of force constant k. The horizontal spring is fixed from one of its ends to a fixed support A. (S) is attached to the other end of the spring and can move without friction on a horizontal surface (Doc. 1).



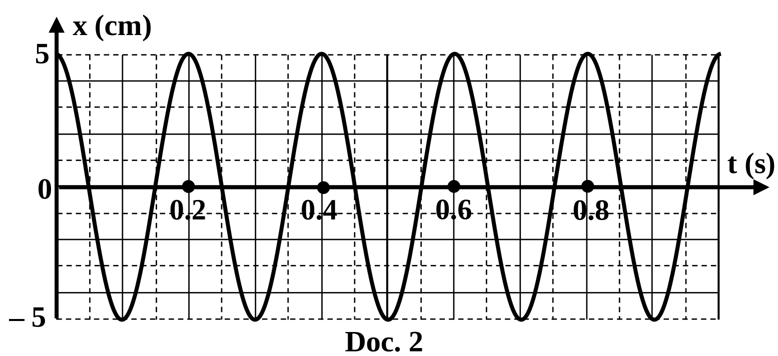
Doc.1

At equilibrium, the center of mass G of (S) coincides with the origin O of the x-axis.
(S) is shifted from its equilibrium position by a displacement x_0 and then it is released without initial velocity at an instant $t_0 = 0$. (S) then performs mechanical oscillations.

At an instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic value of its velocity is $v = x' = \frac{dx}{dt}$.

The aim of this exercise is to determine the maximum speed attained by G.
Take:
- the horizontal plane containing G as the reference level for gravitational potential energy;
- $\pi^2 = 10$.

1) The mechanical energy ME of the system (Oscillator - Earth) is conserved. Why?
2) Write the expression of ME in terms of m, v, k and x.
3) Determine the second order differential equation in x that governs the motion of G.
4) Deduce, in terms of m and k, the expression of the proper (natural) period T_0 of the oscillations.
5) An appropriate device shows x as a function of time (Doc. 2).
5.1) Referring to document 2, indicate the values of T_0 and x_0.
5.2) Deduce the value of k.
5.3) Prove that the mechanical energy of the system (Oscillator - Earth) is $ME = 6.25\times10^{-2}$ J.
5.4) Using document 2, indicate an instant at which the elastic potential energy of the spring is zero.
5.5) Determine the maximum speed attained by G.



Doc. 2

Exercise 2 (6 pts)

Studying the motion of an object

Consider:

- a rail AOB located in a vertical plane composed of two straight parts: a horizontal part AO and an inclined part OB making an angle $\alpha = 30^0$ with the horizontal;
- two objects (S_1) and (S_2) taken as particles of same mass m = 80 g;
- a massless spring (R), of force constant k = 200 N/m and natural length ℓ_0, fixed from one of its ends to a support at A with the other end free.

Take:

- the horizontal plane containing O as the reference level for gravitational potential energy;
- $g = 10\ m/s^2$.

1) Launching particle (S_1)

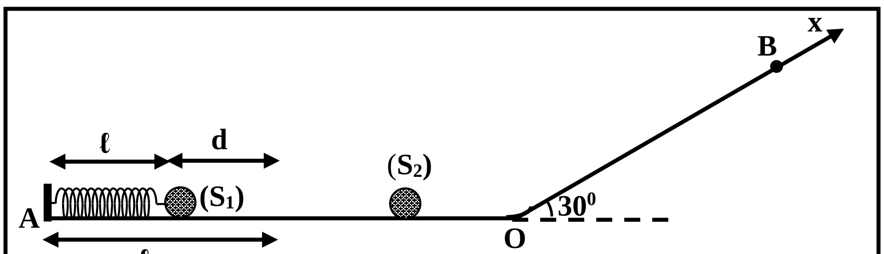
In order to launch (S_1), it is placed against the free end of the spring, the spring is compressed by a distance d, and then the system [Spring - (S_1)] is released from rest (Doc.3).

When the spring returns to its natural length ℓ_0, (S_1) leaves the spring with a velocity $\vec{V}_1$ parallel to AO .

After launching, (S_1) moving with the velocity $\vec{V}_1$, collides head-on with (S_2) which is placed initially at rest on the rail AO.

Just after the collision, (S_1) stops and (S_2) moves with a velocity $\vec{V}_2$ parallel to AO and of magnitude $V_2 = 5$ m/s.

(S_1) and (S_2) move without friction on the rail AO.



Doc. 3

1.1) Apply the law of conservation of linear momentum to show that the magnitude of $\vec{V}_1$ is $V_1 = 5$ m/s.

1.2) Deduce that the collision between (S_1) and (S_2) is elastic.

1.3) Determine the value of d.

2) Motion of (S_2) on the inclined part OB

At the instant $t_0 = 0$, (S_2) starts from O on the inclined part OB with a velocity $\vec{V}_0 = V_0\,\vec{i} = 5\,\vec{i}$ (m/s), where $\vec{i}$ is the unit vector along the x-axis parallel to OB. On this part, (S_2) is submitted to a friction force $\vec{f}$ of constant magnitude f and of direction opposite to its motion.

2.1) Name the external forces acting on (S_2) during its motion along the track OB.

2.2) Show that the sum of the external forces acting on (S_2) during its upward motion along OB is:

$\Sigma\vec{F} = -(f + mg\sin\alpha)\,\vec{i}$.

2.3) The expression of the linear momentum of (S_2) during its upward motion along OB is:

$\vec{P} = (-0.9\,t + 0.4)\,\vec{i}$ (SI).

Knowing that $\frac{d\vec{P}}{dt} = \Sigma\vec{F}$, determine f.

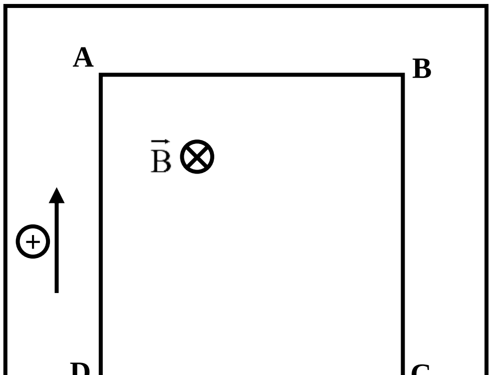
Exercise 3 (7 pts)

Electromagnetic induction

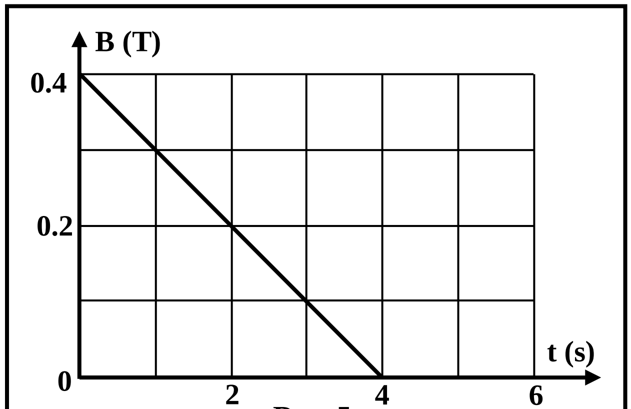
The aim of this exercise is to determine the direction of the induced current in a square-shaped loop by two methods.

For this aim, consider a square-shaped loop ABCD, of side a = 10 cm and resistance r = 2 Ω, is placed in a uniform magnetic field $\vec{B}$, whose magnitude varies with time. The direction of $\vec{B}$ is perpendicular to the plane of the loop (Doc. 4).

Document 5 shows, during the time interval [0, 4 s], the magnitude B of $\vec{B}$ as a function of time.



Doc. 4



Doc. 5

1) An induced current flows in the loop during the time interval [0, 4 s]. Justify.

2) Apply Lenz's law to specify the direction of the induced current in the loop.

3) Prove that the expression of B during the time interval [0, 4 s] is: $B = -0.1t + 0.4$ (SI).

4) Take into consideration the chosen positive direction indicated on document 4, determine, as a function of time, the expression of the magnetic flux crossing the loop.

5) Deduce the value of the induced electromotive force « e ».

6) The induced current in the loop is given by $i = \frac{e}{r}$. Deduce the value and the direction of i.

7) Compare the direction of the induced current obtained in part 6 with that obtained in part 2.

مسابقة في مادة الفيزياء
أسس التصحيح

Exercise 1 (7 pts) **Mechanical oscillations**

Part		Answer	Mark
1		Friction is negligible, then the mechanical energy of the system is conserved. (Or the sum of the works done by the non-conservative forces is zero, then the mechanical energy is conserved).	0.25
2		$ME = KE + EPE = \frac{1}{2} m v^2 + \frac{1}{2} kx^2$	0.5
3		ME = constant , then $\frac{dME}{dt} = 0$, so $m v v' + k x x' = 0$, hence $v (m x'' + k x) = 0$ $v = 0$ is rejected , then $x'' + \frac{k}{m} x = 0$	1
4		The differential equation is of the form: $x'' + \omega_0^2 x = 0$, with $\omega_o = \sqrt{\frac{k}{m}}$ $T_0 = \frac{2\pi}{\omega_o}$; therefore, $T_0 = 2\pi\sqrt{\frac{m}{k}}$	1.5
5	5.1	$T_0 = 0.2$ s and $x_0 = 5$ cm	1
	5.2	$0.2 = 2\pi\sqrt{\frac{0.05}{k}}$ k = 50 N/m	1
	5.3	When the speed is zero, the elongation is maximum, then: $ME = KE + EPE = 0 + EPE = \frac{1}{2} kX^2_{max}$ $ME = 0.5 \times 50 \times 0.05^2 = 0.0625 \text{ J} = 6.25 \times 10^{-2} \text{ J}$	0.75
	5.4	t = 0.05 s **or** t = 0.15 s **or** t = 0.25 s **…..**	0.25
	5.5	When G passes through O, its speed is maximum. Then: $ME = KE + EPE = KE + 0 = \frac{1}{2} mV^2_{max}$ $0.0625 = 0.5 \times 0.05 \times (V)_{max}^2$; therefore, $V_{max} = 1.58$ m/s	0.75

Exercise 2 (6 pts) **Study the motion of an object**

Part		Answer	Mark
1	1.1	$\vec{P}_{J.B.C} = \vec{P}_{J.A.C}$ $m\vec{V}_1 + \vec{0} = \vec{0} + m\vec{V}_2$, $\vec{V}_1 = \vec{V}_2$ then, $V_1 = 5$ m/s	**1.5**
	1.2	System [(S_1), (S_1)] The collision is elastic if $KE_{system(before)} = KE_{system(after)}$ $KE_{(before)} = KE_{(S1)} + KE_{(S2)} = \frac{1}{2}mV_1^2 + 0 = \frac{1}{2} \times 0.08 \times 5^2 + 0 = 1$ J $KE_{(after)} = KE_{(S1)} + KE_{(S2)} = 0 + \frac{1}{2}mV_2^2 = 0 + \frac{1}{2} \times 0.08 \times 5^2 = 1$ J Therefore, the collision is elastic.	**1**
	1.3	Apply the law of conservation of mechanical energy of the system [Oscillator-Earth] $ME_{(R)\text{ is compressed by d}} = ME_{(R)\text{ is in its initial length}}$, $(KE + GPE + EPE)_{(R)\text{ is compressed by d}} = (KE + GPE + EPE)_{(R)\text{ is in its initial length}}$ $0 + \frac{1}{2}kd^2 + 0 = \frac{1}{2}mV_1^2 + 0 + 0$, $\frac{1}{2} \times 200 \times d^2 = \frac{1}{2} \times 0.08 \times 5^2$ then d = 0.1 m = 10 cm	**1.5**
2	2.1	The forces acting on (S_2) on OB are: $m\vec{g}$: its weight, $\vec{N}$: Normal reaction $\vec{f}$: friction	**0.75**
	2.2	$\Sigma\vec{F} = m\vec{g} + \vec{N} + \vec{f}$, Component along the direction $\overrightarrow{Ox}$: $\Sigma\vec{F} = -\, mg\sin\alpha\, \vec{i} + 0\, \vec{i} - f\, \vec{i}$ $\Sigma\vec{F} = -(f + mg\sin\alpha)\vec{i}$ Or : $\Sigma\vec{F} = m\vec{g} + \vec{N} + \vec{f} = -\, mg\sin\alpha\, \vec{i} + mg\cos\alpha\, \vec{j} - N\, \vec{j} - f\, \vec{i}$ But : $mg\cos\alpha\, \vec{j} - N\, \vec{j} = 0$, then, $\Sigma\vec{F} = -(f + mg\sin\alpha)\vec{i}$	**0.75**
	2.3	$\frac{d\vec{P}}{dt} = \Sigma\vec{F}$, $-0.9\, \vec{i} = -(f + mg\sin\alpha)\vec{i}$ $-0.9 = -f - 0.08 \times 10 \times 0.5$ Therefore, f = 0.5 N	**0.5**

Exercise 3 (7 pts) **Electromagnetic induction**

Part	Answer	Mark
1	During the interval [0 s, 4 s], B varies with time, then the magnetic flux varies with time, therefore an emf (e) is induced in the circuit. The circuit is closed, then a current is induced in the circuit.	**1**
2	During the interval [0 s, 4 s], B decreases with time, then the direction of the induced magnetic field as that of $\vec{B}$ to oppose this decrease (Lenz's law). Using the right hand rule, the induced current flows in the loop in the positive direction (clockwise).	**1**
3	In the interval [0s, 4s], B(t) varies linearly with time : B = at + b a = slope = $\frac{0-0.4}{4-0} = -0.1$ T/s 0 = -0.1× 4 + b b = 0.4 T then B = - 0.1t + 0.4	**1**
4	$\phi = BS\cos(\vec{B},\vec{n}) = (-0.1t + 0.4)\times(0.1)^2\times\cos(0)$ $\phi = -10^{-3}\,t + 4\times10^{-3}$ (SI)	**1**
5	$e = -\frac{d\phi}{dt} = 10^{-3}$ V	**1**
6	$i = \frac{e}{r} = \frac{10^{-3}}{2} = 0.5\times10^{-3}$ A i > 0 , then the induced current flows in the positive direction (clockwise).	**1.5**
7	They are the same.	**0.5**