وزارة التربية والتعليم العالي المديريّة العامّة للتربية دائرة الامتحانات الرسميّة	امتحانات الشهادة الثانوية العامة فرع علوم الحياة	دورة العام 2020 الخاصّة الخميس 24 أيلول 2020

مسابقة في مادة الفيزياء المدة: ساعتان	الاسم: الرقم:

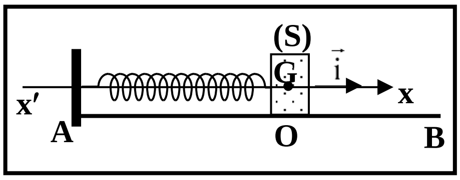
<u>This exam is formed of three obligatory exercises in three pages.</u>
<u>The use of non-programmable calculators is allowed.</u>

Exercise1 (7 points) Mechanical oscillator

Consider a mechanical oscillator constituted of an object (S) of mass m and a spring of negligible mass and stiffness k.

The aim of this exercise is to determine m and k.

The spring, placed horizontally, is fixed from one of its extremities to a fixed support and (S) is attached to the other extremity. (S) may slide without friction on a horizontal rail AB and its center of inertia G can move along a horizontal x- axis.



Doc.1

At equilibrium, G coincides with the origin O of the x-axis (Doc. 1).

At the instant $t_0 = 0$, G has an abscissa x_o and a velocity $\overrightarrow{v_0} = v_0\,\vec{i}$ .Thus, (S) performs mechanical oscillations of amplitude X_m.

At an instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic value of its velocity is $v = \dfrac{dx}{dt}$.

The horizontal plane containing G is considered as the reference level for gravitational potential energy.

1) Specify the type of oscillation of G.

2) Write, at instant t, the expression of the mechanical energy ME of the system [(S), spring, Earth] in terms of x, m, k and v.

3) Establish the second order differential equation in x that governs the motion of G.

4) Deduce, in terms of m and k, the expression of the proper (natural) period T_0 of the oscillations.

5) A solution of the obtained differential equation is: $x = 3 \sin (2.5\pi\, t)$; (x in cm and t in s).

5-1) Write, as a function of t, the expression of v.

5-2) Indicate the value of X_m.

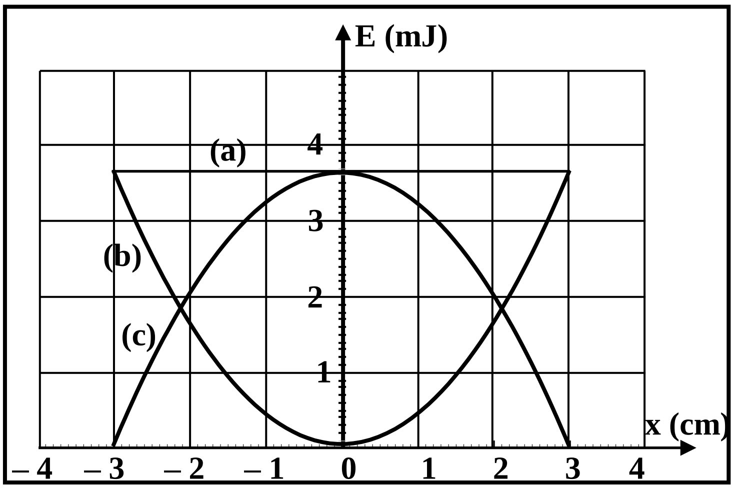
5-3) Calculate the values of x_0 and v_0.

5-4) Deduce the position of G and the direction of its displacement at $t_0 = 0$.

6) The curves (a), (b) and (c) of document 2 represent the kinetic energy KE of (S), the elastic potential energy PE_e of the spring and the mechanical energy ME of the system [(S), spring, Earth].

6-1) Match each curve to the appropriate energy. Justify.

6-2) Using document 2, determine the values of m and k.



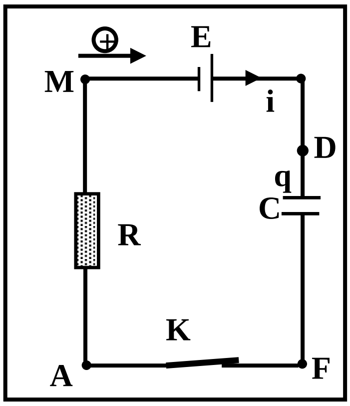
Doc. 2

Exercise 2 (6 points) Charging of a capacitor

The aim of this exercise is to determine the capacitance C of a capacitor.
For this aim, we set-up the series circuit of document 3 that includes:

- an ideal battery (G) of emf E;
- a resistor of resistance R = 1 kΩ;
- a capacitor, initially uncharged, of capacitance C;
- a switch K.

We close the switch K at the instant $t_0 = 0$, and the charging process starts.
At an instant t, plate D of the capacitor carries a charge q and the circuit carries a current i.
An oscilloscope, conveniently connected, allows to display the voltage $u_{AM} = u_R$ across the resistor.



Doc. 3

1) Redraw the circuit of document 3 and show on it the connections of the oscilloscope.

2) Establish the differential equation that governs the variation of the voltage $u_{DF} = u_C$.

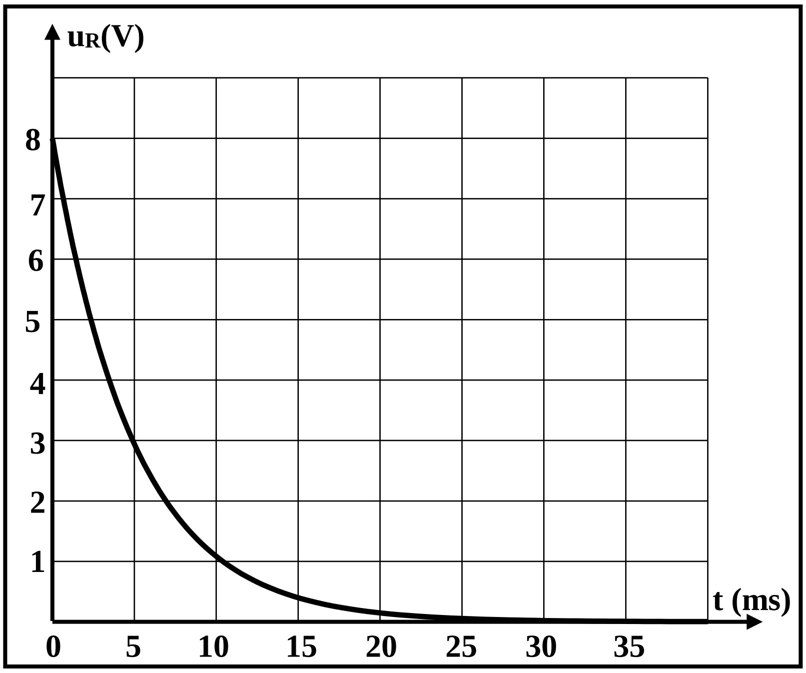
3) Show that $u_C = E\left(1 - e^{\frac{-t}{RC}}\right)$ is a solution of the established differential equation.

4) Deduce the expression of u_R in terms of E, R, C and t.

5) Document 4 shows u_R as a function of time.

5-1) Show that the shape of the curve is in agreement with the expression of u_R.

5-2) Specify the value of E.



Doc. 4

6) The time constant τ of the (R-C) series circuit is given by τ = RC. Choose, from the four statements below, two statements that describe correctly τ during the charging phase of the capacitor. Justify your answer.

Statement 1: τ is the time during which the voltage across the resistor is 37% of its maximum value.

Statement 2: τ is the time during which the voltage across the resistor attains its maximum value.

Statement 3: τ is a physical quantity that permits to slow down the establishment of the steady state.

Statement 4: τ is the time during which the voltage across the capacitor will be equal to that across the resistor.

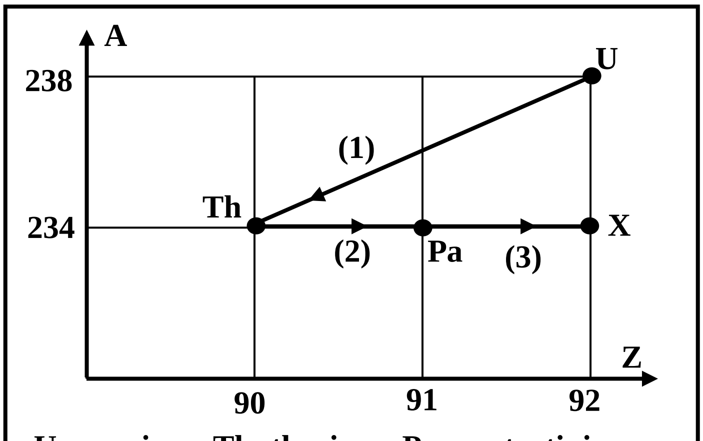
7) Using document 4, determine the value of τ.

8) Deduce the value of C.

Exercise 3 (7 points) The radioactive family of uranium 238

Uranium 238 is a radioactive nuclide which decays to give a daughter nucleus which is radioactive; this nucleus disintegrates into another daughter nucleus, which can be also radioactive, and so on… These successive disintegrations will stop when the obtained daughter nucleus is stable. The set of these disintegrations constitutes a decay family (or series). A radioactive family is given the name of the first element constituting it.

The first four nuclei of the radioactive family of uranium 238 are given in document 5.



Doc. 5

Given:

Planck's constant $h = 6.6 \times 10^{-34}$ J.s;

$1\text{MeV} = 1.6 \times 10^{-13}$J;

the speed of light in air $c = 3 \times 10^{8}$ m/s.

1) Specify the type of decay (α or β^-) for each of the disintegrations (1), (2) and (3) in document 5.

2) X and U are two nuclides of the same chemical element. Justify.

3) Disintegration (1) of uranium 238 is sometimes accompanied with the emission of a γ-ray.
The emitted particle during this disintegration, sometimes has a kinetic energy KE_1= 4.147 MeV and sometimes has a kinetic energy KE_2 = 4.195 MeV. We suppose that uranium 238 is at rest and the kinetic energy of thorium 234 is negligible.

3-1) Indicate the cause of emission of a γ-ray.

3-2) Indicate the value of the kinetic energy of the emitted particle when this disintegration isn't accompanied with the emission of a γ-ray.

3-3) Deduce the energy of the γ-ray that accompanied the disintegration of uranium 238.

3-4) Calculate the wavelength λ_1 of the corresponding radiation.

4) The decay constant of uranium-238: $\lambda_2 = 4.9 \times 10^{-18}$ s^{-1}.

4-1) Calculate, in year, the half-life T of uranium 238.

4-2) Deduce why uranium 238 remains on Earth to the present days.

5) Uranium 238 can be found in some minerals. The radioactive activity of uranium 238 in a mineral sample is, at $t_0 = 0$, A_0 = 8000 Bq.

5-1) Determine the number N_0 of uranium nuclei in this sample at $t_0 = 0$.

5-2) Show, by calculation, that N_0 remains almost the same at t_1 = 100 years and at t_2 = 1000 years.

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امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

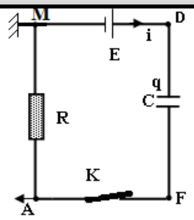
دورة العام 2020 الخاصة
الخميس 24 أيلول 2020

مسابقة في مادة الفيزياء — الاسم:
المدة: ساعتان — الرقم:

Exercise1 (7 points) **Mechanical oscillator**

Part		Answers	Note
1		Free un-damped mechanical oscillations , since there is no friction	0.5
2		$ME = KE + EPE = \frac{1}{2} m v^2 + \frac{1}{2} kx^2$	0.5
3		The oscillations are simple harmonic : ME = constant $\frac{dME}{dt} = 0$; $mvx'' + kxv = 0$; $v= 0$ is rejected, then $x'' + \frac{k}{m}x = 0$	0.75
4		The differential equation has the form: $x'' + \omega_0^2 x = 0$; $\omega_0=\sqrt{\frac{k}{m}}$; $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$	0.5
5	5-1	$v = x' = 3 \times 2.5\pi \cos(2.5\pi t) = 7.5\pi \cos(2.5\pi t)$ $= 23.56 \cos(2.5\pi t)$; v (cm/ s) & t (s)	0.5
	5-2	$X_m = 3$ cm	0.25
	5-3	At t = 0, $x_0 = 3 \sin(0) = 0$ $v_0 = 23.56 \cos(0) = 23.56$ cm/s	0.5 0.5
	5-4	G is at O since $x_0 = 0$ and it moves in the positive direction since $v_0 > 0$	0.25
6	6-1	Curve (a): ME, the oscillations are simple harmonic so ME = constant, Curve (b): EPE, since at t = 0 the object (S) is at O, so x = 0 and then EPE = 0 Curve (c) : KE, since when (S) at $x = X_m$ the speed of (S) is zero so KE = 0	0.5 0.5 0.5
	6-2	For $x = X_m = 0.03$ m ; $EPE = \frac{1}{2} k X_m^2 = 3.6$ mJ $= 3.6 \times 10^{-3}$ J ; k = 8 N/m For x = 0 ; $v = V_m = 23.26$ cm/s $= 23.26 \times 10^{-2}$ m/s ; $KE = \frac{1}{2} m V_m^2 = 3.6$ mJ $= 3.6 \times 10^{-3}$ J ; m = 0.13 kg	0.5 0.75

Exercise 2 (6 points) Charging of a capacitor

Part	Answers	Note
1		0.25
2	$i = \frac{dq}{dt}$, then $i = C\frac{du_C}{dt}$ $u_{DM} = u_{DF} + u_{FA} + u_{AM}$, thus $E = u_C + R\,i$ $E = u_C + R\,C\frac{du_C}{d\,t}$, then $\frac{du_C}{d\,t} + \frac{u_C}{RC} = \frac{E}{RC}$.	1
3	$u_c = E\left(1 - e^{-\frac{t}{R.C}}\right)$, so $\frac{d\,u_c}{d\,t} = \frac{E}{RC}\,e^{-\frac{t}{R.C}}$, replace in the differential equation: $\frac{E}{RC}\,e^{-\frac{t}{RC}} + \frac{E\left(1 - e^{-\frac{t}{R.C}}\right)}{R\ C} = \frac{E}{R\,C}$ $\frac{E}{R\,C} = \frac{E}{R\,C}$ so $u_c(t) = E\left(1 - e^{-\frac{t}{R.C}}\right)$ is a solution	0.75
4	$u_R = Ri = R\,C\frac{du_C}{dt} = R\,C\,\frac{E}{RC}\,e^{-\frac{t}{R.C}} = E\,e^{-\frac{t}{R.C}}$	0.5
5-1	$u_R = Ee^{-\frac{t}{R.C}}$ is a decreasing exponential function, from E to 0, which is in agreement with the graph of document 4	0.5
5-2	$\left.\begin{array}{l}\text{from the equation: at } t_0 = 0 :\ u_R = E\,e^0 = E \\ \text{from the curve : at } t_0 = 0 :\ u_R = 8\text{ V}\end{array}\right\}$ E = 8 V	0.5
6	**Statement 1** : at $t = \tau$; $u_R = E\,e^{-\frac{\tau}{R.C}} = E\,e^{-1} = E \times 0.367 \approx 37\ \%\ E$ **Statement 3:** The steady state is attained after a time $t = 5\ \tau$;therefore, if we increase τ, the duration $5\ \tau$ increases, which slows down the establishment of the steady state.	0.75 0.75
7	a $t = \tau$; $u_R = 0.37 \times 8 = 2.96$ V, from the curve $\tau = 5$ ms	0.5
8	$\tau = R\,C$, then $C = \frac{\tau}{R} = 5 \times 10^{-6}$ F = 5 µF	0.5

Exercise 3 (7 points) **The radioactive family of uranium 238**

Part		Answers	Note
1		Disintegration (1) : ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$ then type α Disintegration (2) : ${}^{234}_{90}Th \rightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e$ then type β^- Disintegration (3) : ${}^{234}_{91}Pa \rightarrow {}^{234}_{92}X + {}^{0}_{-1}e$ then type β^-	**0.5** **0.5** **0.5**
2		Because they have the same charge number Z, they are isotopes of uranium.	**0.25**
3	3-1	The de-excitation of the daughter nucleus Th.	**0.25**
	3-2	When their isn't emission of gamma ray, kinetic energy of the emitted particle is KE_2 = 4.195 MeV	**0.25**
	3-3	When γ ray accompanied disintegration of uranium 238, kinetic energy of the emitted particle decreases, this decrease is due to the energy of the emitted gamma ray. So E_γ = 0.048 MeV	**0.25** **0.25**
	3-4	$E_\gamma = \frac{h.c}{\lambda_1}$; $\lambda_1 = \frac{h.c}{E} = \frac{6.6\times10^{-34}\times3\times10^{8}}{0.048\times1.6\times10^{-13}} = 2.57\times10^{-11}$ m	**1**
4	4-1	$T = \frac{\ln 2}{\lambda_2} = \frac{\ln 2}{4.9\times10^{-18}} = 1.47\times10^{17}s \approx 4.6\times10^{9}$ years	**1**
	4-2	The half-life of uranium-238 is of the order of billions of years, therefore this nuclide needs a lot of time to disintegrate which explains its presence on Earth until today.	**0.5**
5	5-1	$A_0 = \lambda_2 N_0$; $N_0 = \frac{A_0}{\lambda_2} = \frac{8000}{4.9\times10^{-18}} = 1.63\times10^{21}$ nuclei	**1**
	5-2	$N = N_0 e^{-\lambda_2 t}$ t_1 = 100 years: $N = 1.63\times10^{21} e^{-4.9\times10^{-18}\times100\times365\times14\times3600} \approx 1.63\times10^{21}$ t_2 =1000 years: $N = 1.63\times10^{21} e^{-4.9\times10^{-18}\times1000\times365\times14\times3600} \approx 1.63\times10^{21}$	**0.25** **0.25** **0.25**