مسابقة في: مادة الفيزياء	الاسم:
المدّة: ساعتان	الرقم:

This exam is formed of three exercises in 3 pages.
The use of a non-programmable calculator is recommended.

Exercise 1 (7 points)

Characteristics of a coil and a capacitor

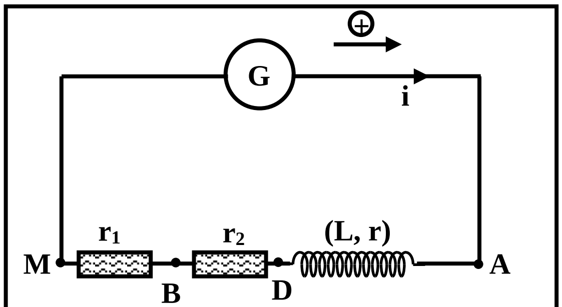
Consider:

- a generator G delivering an alternating sinusoidal voltage:
  $u_{AM} = u_G = U_m \cos(\omega t)$ (SI units);
- a coil of inductance L and resistance r;
- a capacitor of capacitance C;
- two resistors of resistances $r_1 = 10\ \Omega$ and $r_2 = 32\ \Omega$;
- an oscilloscope;
- connecting wires.

The aim of this exercise is to determine L, r and C.

1) Experiment 1

We set-up the circuit of document 1. The circuit thus carries an alternating sinusoidal current i. The oscilloscope, conveniently connected, allows us to display the voltage u_{AM} across the generator on channel (Y_1) and the voltage $u_{BM} = u_{r_1}$ across the resistor r_1 on channel (Y_2).

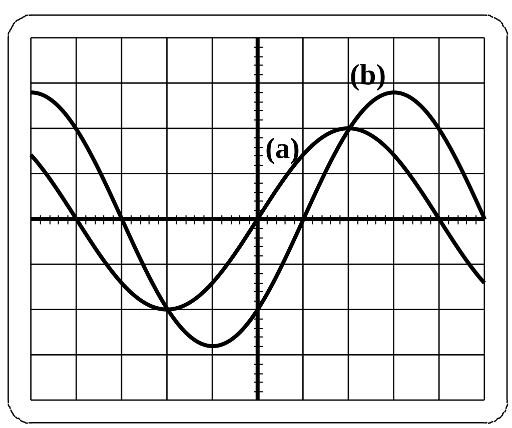


Doc. 1

The obtained waveforms are shown in document 2.

The adjustments of the oscilloscope are:

- vertical sensitivity on (Y_1): $S_{V1} = 5$ V/div;
- vertical sensitivity on (Y_2): $S_{V2} = 0.5$ V/div;
- horizontal sensitivity: $S_h = 2.5$ ms/div.



Doc. 2

1-1) Redraw the circuit of document 1 and show on it the connections of the oscilloscope.

1-2) The waveform (a) represents u_{AM}. Justify.

1-3) Referring to document 2, determine:

1-3-1) the angular frequency ω of the voltage u_{AM};

1-3-2) the amplitudes U_m and U_{m1} of the voltages u_{AM} and u_{BM} respectively;

1-3-3) the phase difference φ between u_{AM} and u_{BM}.

1-4) Write the expression of the voltage u_{BM} as a function of time.

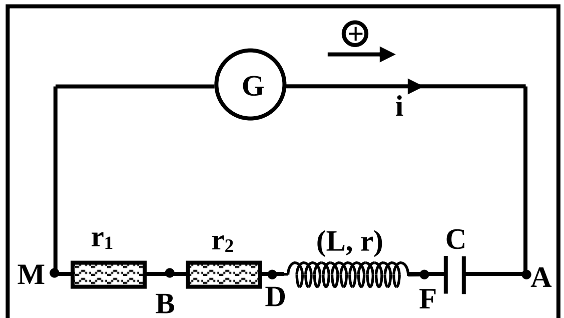
1-5) Deduce the expression of the current i as a function of time.

1-6) Determine the values of L and r by applying the law of addition of voltages and by giving t two particular values.

2) Experiment 2

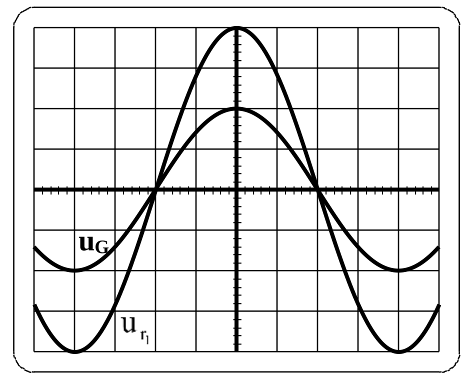
The capacitor is connected in series with the electric components of the circuit of document 1 (Doc. 3).

The oscilloscope, conveniently connected, allows us to display the voltage u_{AM} on channel (Y_1) and the voltage u_{BM} on channel (Y_2). The obtained waveforms are represented in document 4.



Doc. 3

- **2-1)** The circuit is the seat of current resonance. Justify.
- **2-2)** In case of current resonance, the angular frequency ω of the generator is equal to the proper angular frequency ω_0 of the circuit.
  Choose, from the statements below, the one that describes correctly the proper angular frequency ω_0 of the circuit in document 3:
  Statement 1: the proper angular frequency of the circuit is the angular frequency of u_G such that the current i and the voltage u across the coil are in phase.
  Statement 2: the proper angular frequency of the circuit is the angular frequency of u_G such that the amplitude I_m of the current i attains a maximum value.
  Statement 3: the proper angular frequency of the circuit is the angular frequency of u_G such that the amplitude of the voltage across the coil attains a maximum value.
- **2-3)** Write the relation among L, C and ω_0. Calculate C.

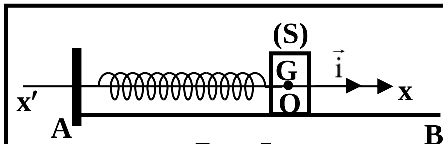


Doc. 4

<u>Exercise 2</u> (6.5 points)

Mechanical oscillator

Consider a mechanical oscillator formed of a spring, of negligible mass and spring constant k, and an object (S) of mass m.
The aim of this exercise is to determine k and m.
The spring is placed horizontally, connected from one of its extremities to a fixed support. (S) is attached to the other extremity of the spring and it may slide without friction on a horizontal rail AB and its center of mass G can move along a horizontal x-axis.
At equilibrium, G coincides with the origin O of the x-axis (Doc. 5).



Doc. 5

(S) is shifted from its equilibrium position and then released without initial velocity at the instant $t_0 = 0$. Thus, (S) performs mechanical oscillations.

At an instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic value of its velocity is $v = \frac{dx}{dt} = x'$.

The horizontal plane containing G is considered as a reference level for gravitational potential energy.

1) The differential equation that describes the motion of G is: $2x'' + 200x = 0$ (SI units).
Use this differential equation to:
- **1-1)** show that the motion of G is simple harmonic;
- **1-2)** calculate the value of the proper angular frequency ω_0 of oscillations.

2) The time equation of the motion of G is of the form: $x = X_m \cos(\omega_0 t)$, where X_m is the amplitude of x.
- **2-1)** Write the expression of v in terms of X_m, ω_0 and t.
- **2-2)** Using the expressions of x and v, show that: $\omega_0^2 = \frac{v^2}{X_m^2 - x^2}$.

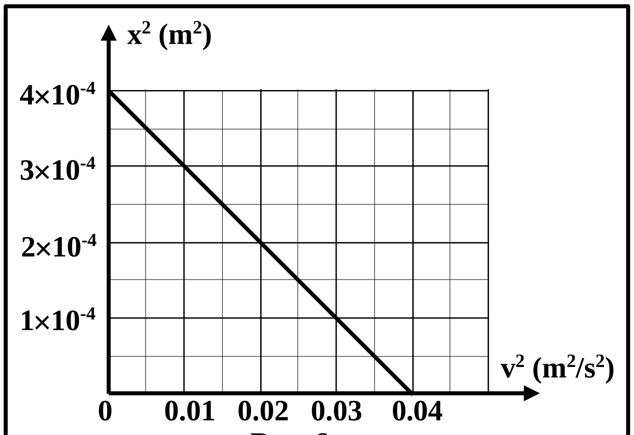
3) Applying the principle of conservation of mechanical energy «ME» of the system [(S), spring, Earth], show that: $x^2 = a v^2 + b$, where «a» and «b» are two constants to be determined in terms of k, m and ME.

4) Document 6 shows x^2 as a function of v^2.
Using document 6:

4-1) calculate X_m;

4-2) calculate again the value of ω_0.

5) Determine the values of k and m knowing that ME = 0.04 J.



Doc. 6

<u>Exercise 3</u> (6.5 points)

Dating of a volcanic rock

Some of the volcanic rocks contain the radioactive isotope of potassium ${}^{40}_{19}K$ of half-life T and radioactive constant λ.

A small proportion of this isotope decays into argon ${}^{40}_{18}Ar$.

The aim of this exercise is to determine the age of a volcanic rock.

1) Indicate the composition of the potassium ${}^{40}_{19}K$ nucleus.

2) The decay equation of potassium-40 into argon-40 is: ${}^{40}_{19}K \rightarrow {}^{40}_{18}Ar + {}^{A}_{Z}X$.

2-1) Determine Z and A, indicating the used laws.

2-2) Name the emitted particle ${}^{A}_{Z}X$.

3) A sample of a volcanic rock contains at the instant of its formation, $t_0 = 0$, N_0 nuclei of potassium-40 that decay into argon-40.

3-1) Write the expression of the remaining number N_K of potassium-40 nuclei in terms of N_0, t and λ.

3-2) Deduce that the number of the formed argon-40 nuclei is: $N_{Ar} = N_0\,(1 - e^{-\lambda t})$.

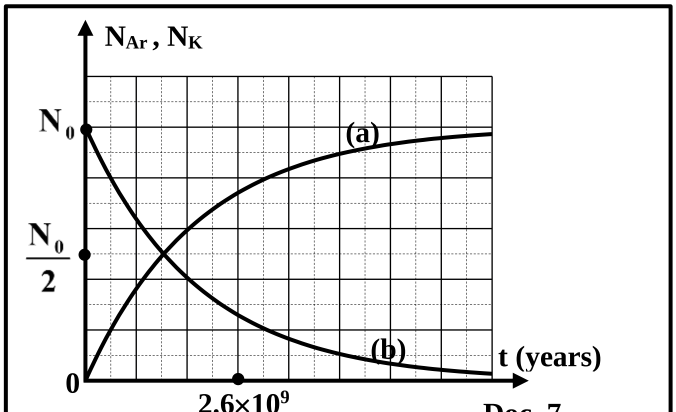
3-3) Determine, in terms of λ, the expression of t when $N_{Ar} = N_K$.

4) The curves (a) and (b) of document 7 represent N_K and N_{Ar} as functions of time.

4-1) Specify the curve that represents N_K.

4-2) Determine graphically the half-life T of potassium-40.

4-3) Deduce the value of λ.



Doc. 7

5) The sample of the volcanic rock contains at the instant of its formation, $t_0 = 0$, N_0 nuclei of potassium-40 that decay into argon-40. At this instant the sample does not contain any argon-40 nucleus.
At an instant t:

- N_K is the remaining number of nuclei of N_0 of potassium-40;
- N_{Ar} is the formed number of the argon-40 nuclei.

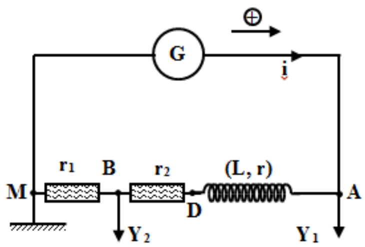
A geologist analyzes this sample to determine the age of the volcanic rock. He finds that the number N_{Ar} of argon-40 nuclei is 3 times the number N_K of potassium-40 nuclei.

5-1) Show that $\frac{N_0}{N_K} = 4$.

5-2) Deduce the age of the rock.

<u>Exercise 1</u> (7 points)

Characteristics of a coil and a capacitor

Part			Answer	Mark
1	1.1			0.5
	1.2		In the R-L series circuit, u_G leads i. Since curve (a) leads curve (b) , then it represents u_{AM}.	0.5
	1.3	1	$T = S_h \times x = 2.5 \times 8 = 20$ ms $= 20 \times 10^{-3}$ s then $\omega = \frac{2\pi}{T} = \frac{2\pi}{20\times10^{-3}} = 100\,\pi$ rad/s	0.75
		2	$U_m = S_{v1} \times y_1 = 2 \times 5 = 10$ V $U_{m1} = S_{v2} \times y_2 = 2.8 \times 0.5 = 1.4$ V	0.75
		3	$\varphi = \frac{2\pi \times d}{D} = \frac{2\pi \times 1 \text{ div}}{8 \text{ div}} = \frac{\pi}{4}$ rad	0.5
	1.4		u_{AM} leads u_{BM} by $\frac{\pi}{4}$ rad. $u_{BM} = 1.4 \cos(100\,\pi t - \frac{\pi}{4})$ (u_{BM} in V and t in s)	0.5
	1.5		$U_{BM} = r_1 \times i$, then $i = \frac{u_{BM}}{r_1} = 0.14 \cos(100\,\pi t - \frac{\pi}{4})$ (i in A and t in s)	0.5
	1.6		$u_{AM} = u_{AD} + u_{DB} + u_{BM}$ $U_m \cos(\omega t) = ri + L\frac{di}{dt} + r_2\, i + r_1 i$ $U_m \cos(\omega t) = r \times 0.14 \cos(100\,\pi t - \frac{\pi}{4}) + L\,[-14 \sin(100\,\pi\, t - \frac{\pi}{4})] + (r_2 + r_1)\, 0.14 \cos(100\,\pi\, t - \frac{\pi}{4})$	0.5
			For $t = \frac{\pi}{4\omega}$ ($\omega t = \frac{\pi}{4}$) : $U_m \frac{\sqrt{2}}{2} = r \times 0.14 + 0 + (r_2 + r_1)\, 0.14$ $5\sqrt{2} = 0.14\, r + 42 \times 0.14$; we calculate $r = 8{,}5\ \Omega$	0.5
			For $\omega t = 0$: $U_m = r \times 0.14 \times \frac{\sqrt{2}}{2} + 14\, L\pi \frac{\sqrt{2}}{2} + (r_2 + r_1)\, 0.14 \frac{\sqrt{2}}{2}$ $10 = 8.5 \times 0.14 \times \frac{\sqrt{2}}{2} + 14\, L\pi \frac{\sqrt{2}}{2} + 42 \times 0.14 \frac{\sqrt{2}}{2}$ we calculate $L = 0{,}16$ H	0.5
2	2.1		u_G and u_{r1} are in phase, with u_1 is the image of i.	0.25
	2.2		Statement 2	0.5
	2.3		In the case of current resonance, we have $\omega_G = \omega_0 = 100\,\pi$ and $LC\omega_0^2 = 1$ Then, $C = 6.33 \times 10^{-5}$ F	0.25 0.5

Exercise 2 **(6.5 points)**

Mechanical oscillator

Part		**Answer**	**Mark**
1	**1.1**	The differential equation $2x'' + 200x = 0$ can be written as: $x'' + 100x = 0$. Then, it has the form of: $x'' + \omega_0^2 x = 0$ This equation governs a simple harmonic motion of G.	**0.75**
	1.2	$\omega_0^2 = 100$; $\omega_0 = 10$ rad/s	**0.5**
2	**2.1**	$x = X_m \cos(\omega_0 t)$ $v = x' = -\omega_0 X_m \sin(\omega_0 t)$	**0.5**
	2.2	$\frac{x^2}{X_m^2} = \cos^2\omega_0 t$ and $\frac{v^2}{\omega_0^2 X_m^2} = \sin^2\omega_0 t$ $\sin^2\omega_0 t + \cos^2\omega_0 t = 1$, then $\frac{x^2}{X_m^2} + \frac{v^2}{\omega_0^2 X_m^2} = 1$ $\omega_0^2 X_m^2 = \omega_0^2 x^2 + v^2$, then $\omega_0^2(X_m^2 - x^2) = v^2$ $\omega_0^2 = \frac{v^2}{X_m^2 - x^2}$, then verified	**0.75**
3		ME = constant, then $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = ME$ $\frac{1}{2}kx^2 = ME - \frac{1}{2}mv^2$ then $x^2 = \frac{2ME}{k} - \frac{mv^2}{k}$ $x^2 = -\frac{m}{k}v^2 - \frac{2ME}{k}$ this equation has the form: $x^2 = av^2 + b$ $a = -\frac{m}{k}$ and $b = \frac{2ME}{k}$	**1.25**
4	**4.1**	$X_m^2 = 4\times10^{-4}$ m^2, then $X_m = 2\times10^{-2}$ m = 2 cm	**0.5**
	4.2	When $x^2 = 0$, $v^2 = 0.04$, then v = 0.2 m/s $\omega_0^2 = 100$ then $\omega_0 = 10$ rad/s	**0.75**
5		At t = 0: $v_0 = 0$, $X_m = 2\times10^{-2}$ m ME = KE + EPE + GPE then: $0.04 = 0 + 0 + \frac{1}{2}kX_m^2$ $k = \frac{2\times0.04}{X_m^2} = \frac{2\times0.04}{4\times10^{-4}} = 200$ N/m When x = 0, $V_m = 0.2$ m/s $ME = \frac{1}{2}mV_m^2$ then: $m = \frac{2\times ME}{V_m^2} = \frac{2\times0.04}{0.04} = 2$ kg **<u>OR:</u>** $b = \frac{2\times ME}{k}$; $x^2 = av^2 + b$ if $v^2 = 0$, then $x^2 = 4\times10^{-4}$ m^2, then $4\times10^{-4} = b = \frac{2\times ME}{k}$ $k = \frac{2\times ME}{4\times10^{-4}} = 200$ N/m $a = -\frac{m}{k}$; $a = \frac{x^2 - x_0^2}{v^2 - v_0^2} = \frac{0-4\times10^{-4}}{0.04-0} = -10^{-2}$ $-10^{-2} = -\frac{m}{200}$ then m = 2 kg	**1.5**

<u>Exercise 3</u> (6.5 points)

Dating of a volcanic rock

Part		Answer	Mark
1		Number of protons Z = 19 Number of neutrons N = A – Z = 40 – 19 = 21	**0.5**
2	**2.1**	According to the law of conservation of mass number : 40 = 40 + A then A = 0 According to the law of conservation of charge number : 19 = 18 + Z then Z = 1	**1**
	2.2	${}^{0}_{1}X = {}^{0}_{1}e$, the emitted particle is positron	**0.25**
3	**3.1**	$N_K = N_0 \times e^{-\lambda t}$	**0.5**
	3.2	$N_{Ar} = N_0 - N_K = N_0 - N_0 \times e^{-\lambda t} = N_0 (1 - e^{-\lambda t})$	**0.5**
	3.3	$N_{Ar} = N_K$ then $N_0 (1 - e^{-\lambda t}) = N_0 \times e^{-\lambda t}$ then $1 - e^{-\lambda t} = e^{-\lambda t}$ then $2 e^{-\lambda t} = 1$, so $e^{\lambda t} = 2$ then $\lambda t = \ln 2$ then $t = \frac{\ln 2}{\lambda}$	**0.75**
4	**4.1**	(b) represents N_K since N_K decreases exponentially as a function of time.	**0.5**
	4.2	When t = T, we have $N_K = \frac{N_0}{2}$. Graphically: $T = \frac{2.6 \times 10^9}{2} = 1.3 \times 10^9$ years	**0.75**
	4.3	$\lambda = \frac{\ln 2}{T} = \frac{0.693}{1{,}3 \times 10^9} = 0.533 \times 10^{-9}$ year^{-1} = 0.016 s^{-1}	**0.5**
5	**5.1**	$N_0 (1 - e^{-\lambda t}) = 3 \times N_0 \times e^{-\lambda t}$ $1 = 3 \times e^{-\lambda t} + e^{-\lambda t} = 4 e^{-\lambda t}$ $e^{\lambda t} = 4$ Then, $N_K = \frac{N_0}{e^{\lambda t}} = \frac{N_0}{4}$. Then, $\frac{N_0}{N_K} = 4$ verified **<u>Or</u>**: $N_K = N_0 - N_{Ar} = N_0 - 3N_K$ then $4 N_K = N_0$, so $\frac{N_0}{N_K} = 4$	**0.5**
	5.2	$\frac{N_0}{N_K} = 4$ $N_0 = 4 \times N_K = 4 \times N_0 e^{-\lambda t}$ $\frac{1}{4} = e^{-\lambda t}$ then $-\lambda t = \ln(0.25)$ then $t = \frac{\ln(0,25)}{-\lambda} = \frac{\ln(0,25)}{-\ln 2 \times T} = 2T = 2.6 \times 10^9$ years **<u>OR :</u>** $N_K = \frac{N_0}{4} = \frac{N_0}{2^2}$. Then, $t = 2T = 2 \times 1.3 \times 10^9 = 2.6 \times 10^9$ years	**0.75**