

الاسم:
الرقم:

مسابقة في مادة الفيزياء
المدّة: ساعتان

This exam is formed of three exercises in three pages
The use of non- programmable calculators is recommended

Exercise 1 (7 points)

Determination of the stiffness of a spring

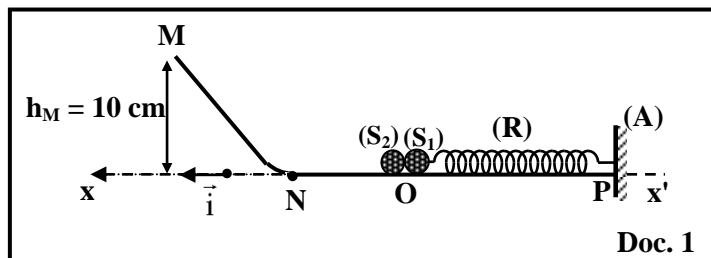
In order to determine the stiffness k of a massless spring (R), we consider:

- a track MNP found in a vertical plane ;
- a massless spring (R) of horizontal axis and stiffness k , having one end fixed to a support (A); the other end is connected to an object (S_1) considered as a particle of mass $m_1 = 0.2$ kg;
- an object (S_2) considered as a particle of mass $m_2 = 0.3$ kg, placed at the origin O of a horizontal x -axis of unit vector \hat{i} (Doc. 1).

Neglect all the forces of friction.

Take:

- the horizontal plane passing through NP as a reference level for gravitational potential energy;
- $g = 10$ m/s².



1- Collision between (S_1) and (S_2)

At equilibrium, (S_1) coincides with O. (S_1) is shifted from O to the right by a certain distance and it is released from rest. (S_1) reaches O with a velocity $\vec{V}_1 = 2 \hat{i}$ (m/s), and enters into a head-on collision with (S_2) initially at rest. Just after collision, (S_1) rebounds with a velocity $\vec{V}'_1 = -0.4 \hat{i}$ (m/s) and (S_2) moves to the left with a velocity $\vec{V}'_2 = V'_2 \hat{i}$.

1-1) Applying the principle of conservation of linear momentum for the system [(S_1), (S_2)], show that $V'_2 = 1.6$ m/s.

1-2) Specify whether this collision is elastic or not.

2- Motion of (S_2) after collision

Just after collision, (S_2) moves along the horizontal track PN with the speed V'_2 and then continues its motion along the inclined plane MN. (S_2) leaves the inclined plane at M with a speed V_M . The height of M above the reference level is $h_M = 10$ cm. Determine the speed V_M of (S_2) at point M.

3- Oscillation of (S_1)

After collision, (S_1) oscillates along the x -axis. At an instant t , the abscissa of (S_1) is x and the algebraic value of its velocity is $v = \frac{dx}{dt}$.

3-1) Write, at an instant t , the expression of the mechanical energy of the system [(S_1), spring, Earth] in terms of m_1 , k , x and v .

3-2) Derive the second order differential equation in x that describes the motion of (S_1).

3-3) Deduce the expression of the proper period T_0 .

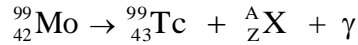
3-4) Calculate k knowing that $T_0 = 0.314$ s.

Exercise 2 (6 points)

Scintigraphy in medicine

The bones scintigraphy is a medical examination that permits to observe bones and articulations. The aim of this exercise is to study a radioactive sample used in this scintigraphy.

This medical examination uses technetium-99 produced due to the disintegration of molybdenum-99 according to the following nuclear reaction:



The energy of the emitted gamma (γ) photon is 140 keV.

Given: $c = 3 \times 10^8 \text{ m.s}^{-1}$; $1\text{eV} = 1.6 \times 10^{-19} \text{ J}$; Planck's constant $h = 6.6 \times 10^{-34} \text{ J.s}$.

- 1- Identify the emitted particle ${}^A_Z\text{X}$, indicating the used laws.
- 2- The emitted particle ${}^A_Z\text{X}$ is always accompanied with the emission of another particle. Name this particle.
- 3- Indicate the cause of the emission of the gamma photon.
- 4- Calculate the wavelength of the emitted gamma photon.
- 5- Technetium-99 is a radioactive substance.

The graph of document 2 represents the activity of a sample of technetium-99 as a function of time.

Using document 2, show that the radioactive period (half-life) of technetium-99 is $T = 6 \text{ hrs}$.

- 6- In a session of scintigraphy examination, a patient is injected at $t_0 = 0$ by technetium-99 of activity $A_0 = 530 \times 10^6 \text{ Bq}$. At the end of the examination session, the activity of technetium in the body of the patient is 63% of its initial value.

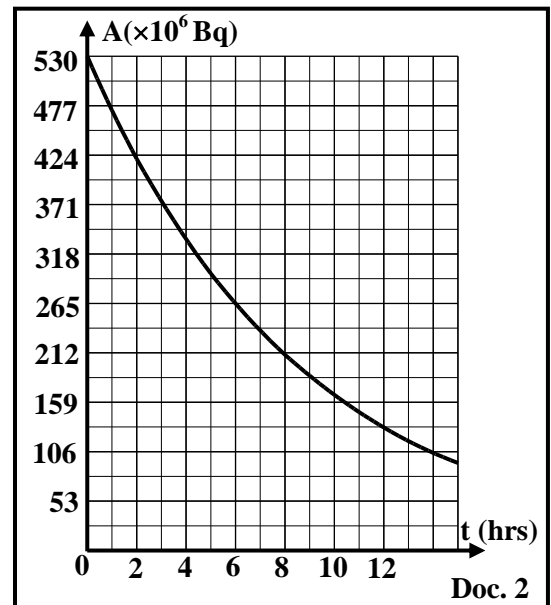
6-1) Write, at instant t , the expression of the activity A in terms of A_0 , t and the decay constant λ .

6-2) Using the preceded expression, determine:

6-2-1) the duration of the examination session;

6-2-2) the ratio $\frac{A}{A_0}$ of technetium-99 at

$$t = 40 \text{ hrs.}$$



Exercise 3 (7 points)

RLC series circuit in the radio

One of the useful applications of an RLC series circuit is used in radios.

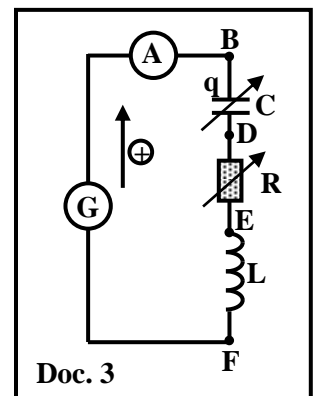
This exercise studies the effect of the capacitance C on the detection of the radio wave and the effect of the resistance R on the loudness of the sound emitted by the radio.

1- Experimental study of an RLC series circuit

Document 3 represents an RLC series circuit formed of:

- a capacitor of adjustable capacitance C ;
- a resistor of adjustable resistance R ;
- a coil of inductance $L = 0.317 \text{ H}$ and negligible resistance;
- an ammeter (A) of negligible resistance.

This circuit is connected across a generator (G) maintains across its terminals an



alternating sinusoidal voltage $u_G = u_{BF} = 3 \sin(\omega t)$, (u_G in V, t in s) and $\omega = 314$ rad/s.

The expression of the current in the circuit is $i = I_m \sin(\omega t + \varphi)$.

For each value of C , the ammeter permits to obtain the amplitude I_m of the current i .

The graph of document 4 represents I_m as a function of C .

1-1) Indicate the value C_0 of C at which I_m attains a maximum value.

1-2) Calculate the value of $LC_0\omega^2$.

1-3) Name then the electric phenomenon observed on document 4.

1-4) The capacitance of the capacitor is $C = 32 \mu\text{F}$.

1-4-1) Pick out graphically the value of I_m .

1-4-2) Show that the expression of the current is given by:

$$i = 0.3 \sin(314 t), \quad (i \text{ in A, } t \text{ in s}).$$

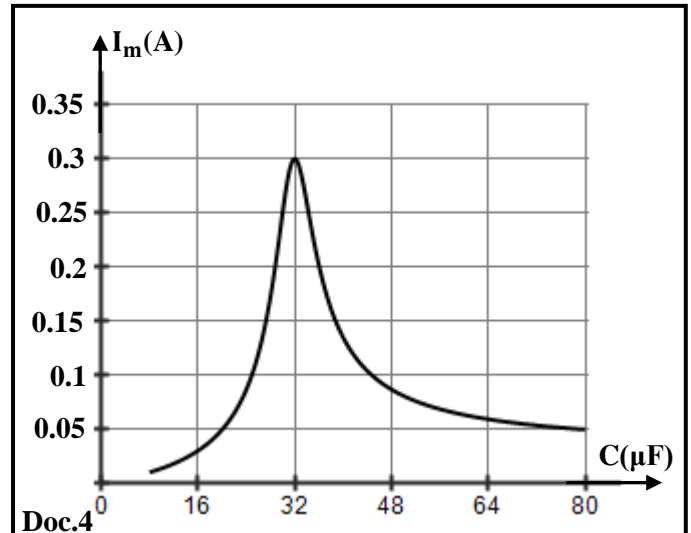
1-4-3) Determine the expression of the voltage $u_L = u_{EF}$ across the terminals of the coil as a function of time t .

1-4-4) Determine the expression of the voltage $u_C = u_{BD}$ across the terminals of the capacitor as a function of time t .

1-4-5) Show that $u_R \sqcup u_G = 3 \sin(314t)$, using the law of addition of voltages $u_G = u_C + u_L + u_R$ where $u_R = u_{DE}$ is the voltage across the resistor.

1-4-6) Deduce the value of R .

1-4-7) We decrease the value of R to 2Ω . Calculate the new value of the maximum current in the circuit using the relation $u_R = u_G$.



2 - RLC series circuit in the radio

Each radio station broadcasts an electromagnetic wave (radio wave) of precise frequency f .

When this radio wave of frequency f is received by the antenna of a radio, it is converted into electric sinusoidal signal of same frequency f ; thus the antenna plays the role of a generator and feeds the RLC series circuit in the radio.

Given:

- the inductance of an RLC series circuit in a radio is $L = 0.2$ mH;
- the values of R and C can be adjusted;
- when the circuit enters an electric phenomenon similar to that of part (1-3) the antenna receives the desired frequency of the wave of the broadcast.

2-1) Determine the value of C so that the antenna receives a radio wave of desired frequency 1000 kHz.

2-2) To increase the intensity of the emitted sound by the radio we have to increase the value of the current in the circuit. Indicate whether we have to increase or decrease the resistance R in order to increase the intensity of the emitted sound by the radio.

Exercise 1 : (7 points)		Determination of stiffness	
Part		Answer	Mark
1	1-1	$\vec{P}_{\text{just before}} = \vec{P}_{\text{just after}}$ $m_1 \vec{V}_1 + \vec{0} = m_1 \vec{V}'_1 + m_2 \vec{V}'_2$ $0.2 \times 2 \vec{i} = 0.2 \times (-0.4) \vec{i} + 0.3 \vec{V}'_2$ $0.48 \vec{i} = 0.3 \vec{V}'_2 ; \vec{V}'_2 = 1.6 \vec{i}, \text{ then } V'_2 = 1.6 \text{ m/s}$	1.25
	1-2	$KE_{\text{before}} = \frac{1}{2} m_1 V_1^2 = \frac{1}{2} (0.2) \times (2)^2 = 0.4 \text{ J}$ $KE_{\text{after}} = \frac{1}{2} m_1 V'^2_1 + \frac{1}{2} m_2 V'^2_2 = \frac{1}{2} (0.2) \times (0.4)^2 + \frac{1}{2} (0.3) \times (1.6)^2 = 0.4 \text{ J}$ $KE_{\text{before}} = KE_{\text{after}}, \text{ then the collision is elastic}$	0.5 0.5 0.5
2		$ME_{(O)} = ME_{(M)}$ (forces of friction are neglected) $KE_{(O)} + PE_{g(O)} = KE_{(M)} + PE_{g(M)}$ $\frac{1}{2} (0.3) \times (1.6)^2 + 0 = 0.3 \times 10 \times 0.1 + \frac{1}{2} (0.3) V_M^2$ $0.348 = 0.3 + 0.15 V_M^2$ $V_M^2 = 0.56, \text{ then } V_M = 0.748 \text{ m/s}$	0.5 0.5 0.5
3	3-1	$ME = KE + PE_g + PE_{el} = \frac{1}{2} m_1 v^2 + \frac{1}{2} k x^2$	0.5
	3-2	No friction : $ME = \text{constant}, \text{ then } \frac{dME}{dt} = 0$ $\frac{1}{2} m_1 2vv' + \frac{1}{2} k 2xx' = 0 ; (v = x' \neq 0, v' = x''), \text{ then } m_1 x'' + kx = 0$ $x'' + \frac{k}{m_1} x = 0$	1
	3-3	The differential equation has the form $x'' + \omega_0^2 x = 0$ $\omega_0^2 = \frac{k}{m_1}$ and $T_0 = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{m_1}{k}}$	0.25 0.5
	3-4	$T_0 = 0.314$ and $T_0 = 2\pi \sqrt{\frac{m_1}{k}}$ $0.314 = 2 \times 3.14 \sqrt{\frac{0.2}{k}}$ then $k = 80 \text{ N/m}$	0.5

Exercise 2: (6 points)		Scintigraphy in medicine		
Part	Answer	Mark		
1	According to the laws of conservation of mass number and charge number : $A = 0$ and $Z = -1$	0.75		
	Then, ${}^A_Z\text{X}$ is an electron of symbol ${}^0_{-1}\text{e}$	0.5		
2	Antineutrino	0.5		
3	The de-excitation of the daughter nucleus "Technetium"	0.5		
4	$E = h \frac{c}{\lambda}, \lambda = \frac{hc}{E}$, then $\lambda = 8.839 \times 10^{-12} \text{ m}$	0.75		
5	At $t_0 = 0$; $A_0 = 530 \times 10^6 \text{ Bq}$ At $t = T$: $A = \frac{A_0}{2} = 265 \times 10^6 \text{ Bq}$ Therefore, $t = T = 6 \text{ hrs}$ (from graph)	0.75		
6	6-1	$A = A_0 e^{-\lambda t}$	0.5	
	6-2	6-2-1	$0.63 A_0 = A_0 e^{-\lambda t}$; $\ln 0.63 = -\lambda t$ $0.46 = \lambda t$ but $\lambda = \frac{\ln 2}{T} = 0.115 \text{ hr}^{-1}$ Then, $t = \frac{0.46}{\lambda} = 4 \text{ hrs}$	1
		6-2-2	$\frac{A}{A_0} = e^{-\lambda t} = e^{-0.115 \times 40} = 0.01 = 1 \%$	0.75

Exercise 3: (7 points)		RLC series circuit		
Part		Answer	Mark	
1	1-1	$C_0 = 32 \mu\text{F}$	0.25	
	1-2	$LC_0\omega^2 = 0.317 \times 32 \times 10^{-6} \times (314)^2 = 1$	0.5	
	1-3	Current resonance	0.5	
	1-4	1	$I_m = 0.3 \text{ A}$	0.25
		2	$i = I_m \sin(\omega t + \varphi) = 0.3 \sin(314 t)$, since $I_m = 0.3 \text{ A}$ and in case of current resonance $\varphi = 0$	0.5
		3	$u_L = L \frac{di}{dt} = L \times 0.3 \times 314 \times \cos(314 t) = 29.86 \cos(314 t)$	0.75
		4	$u_C = u_{BD} = \frac{q}{C}$ $i = \frac{dq}{dt} = C \frac{du_C}{dt}$; $du_C = \frac{i}{C} dt$ Then: $u_C = \frac{0.3}{32 \times 10^{-6}} \int \sin(314 t) dt$ $u_C = \frac{-0.3}{32 \times 10^{-6} \times 314} \cos(314 t) = -29.656 \cos(314 t)$	1
		5	$u_G = u_C + u_L + u_R$ but $u_C \cong -u_L$ then $u_C + u_L = 0$ then $u_G \cong u_R = 3 \sin(314 t)$	0.75
6	$U_{Rm} = 3 = R I_m$, then $R = \frac{3}{0.3} = 10 \Omega$	1		
7	$U_{Rm} = 3 = R I'_m$, then $I'_m = \frac{3}{2} = 1.5 \text{ A}$	0.5		
2	2-1	Current resonance: $f^2 = \frac{1}{4\pi^2 LC}$ Then $C = \frac{1}{4\pi^2 L f^2} = 1.267 \times 10^{-10} \text{ F} = 0.126 \text{ nF}$	0.75	
	2-2	R should be decreased	0.25	