

الدورة العادية للعام 2011	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ساعتان	

**This exam is formed of three exercises in three pages.**  
**The use of non-programmable calculators is recommended.**

### **First exercise (7 points)**

#### **Horizontal elastic pendulum**

The aim of this exercise is to study some physical quantities associated to a horizontal elastic pendulum, formed of a spring of force constant  $K = 20 \text{ N/m}$  and a solid (S) of mass  $m = 500 \text{ g}$ .

Take  $g = 10 \text{ m/s}^2$ ,  $\pi^2 = 10$  and neglect all resistive forces.

#### **A – Theoretical study**

The spring, placed horizontally, is fixed from one extremity to a fixed support. We attach the solid (S) to the other extremity. (S) can move along a horizontal rail CD and its center of inertia G can move along

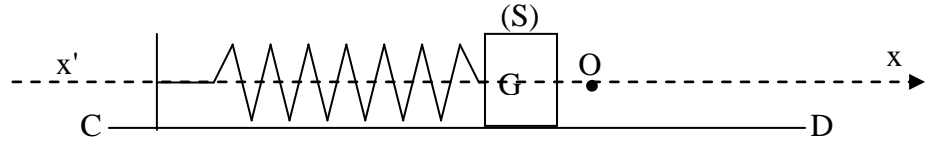


Fig. 1

the horizontal axis  $x'x$ . At equilibrium, G coincides with the origin O of the axis  $x'x$ .

We shift (S) to the left starting from O; G occupies then the position  $G_0$  such that  $x_0 = \overline{OG_0} = -10 \text{ cm}$ . At the instant  $t_0 = 0$ , (S) is released without velocity. At an instant  $t$ , the abscissa of G is  $x$  and the algebraic value of its velocity is  $v = \frac{dx}{dt}$  (Fig. 1).

The horizontal plane passing through G is considered as the reference level for the gravitational potential energy.

- 1) a) Derive the differential equation in  $x$  that describes the motion of G.  
b) i) Deduce the expression of the proper angular frequency  $\omega_0$  of this oscillator and that of its proper period  $T_0$ .  
ii) Calculate  $\omega_0$  and  $T_0$ .
- 2) The time equation  $x = X_m \cos(\omega_0 t + \varphi)$  is the solution of the previous differential equation,  $X_m$  and  $\varphi$  being constants. Determine the values of  $X_m$  and  $\varphi$ .
- 3) a) Determine the expression of  $v$  as a function of time.  
b) Deduce the maximum value of  $v$ .
- 4) Taking into consideration the initial conditions, trace the shape of the curve representing the variation of  $x$  as a function of time.
- 5) a) Calculate the value of the mechanical energy of the system (oscillator, Earth).  
b) Find again the maximum value of  $v$ .

#### **B – Exploitation of the curves of the energies**

An appropriate apparatus provides the curves giving the variation, as a function of time, of the kinetic energy and the elastic potential energy of the system (oscillator, Earth) (Fig. 2).

- 1) Identify, with justification, the two curves a and b.
- 2) The kinetic energy and the elastic potential energy are periodic functions of period  $T$ .  
Determine the relation between  $T$  and  $T_0$ .

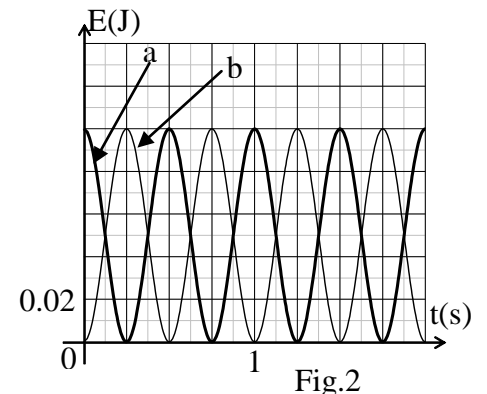
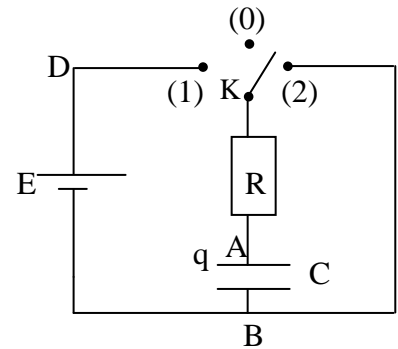


Fig.2

## Second exercise (7 points)

### Discharging of a capacitor: The lightning

The electric circuit of the adjacent figure allows us to perform charging and discharging of a capacitor of capacitance  $C$ , through a resistor of resistance  $R$ . The used generator has a constant electromotive force  $E$  and is of negligible internal resistance.



#### A – Charging of the capacitor

The capacitor is initially uncharged and the switch  $K$  is in position (0).

- 1) To which position, (1) or (2), must we turn the switch  $K$  in order to charge the capacitor?
- 2) The variation of the voltage  $u_C = u_{AB}$  across the terminals of the

capacitor as a function of time is given by the expression:  $u_C = E (1 - e^{-\frac{t}{RC}})$ . Deduce the value of  $u_C$  in terms of  $E$ , at the end of the charging of the capacitor.

#### B – Discharging of the capacitor

The charging of the capacitor being completed, the switch  $K$  is again in position (0).

- 1) To which position must we turn the switch  $K$  in order to discharge the capacitor?
- 2) The instant  $t_0 = 0$  corresponds to the starting of the discharging. At an instant  $t$ , the circuit carries a current  $i$ .
  - a) Draw the circuit of discharging and indicate on it the real direction of the current chosen as a positive direction.
  - b) i) In this case, the current is written as  $i = -\frac{dq}{dt}$  and not as  $i = +\frac{dq}{dt}$ . Why?
    - ii) Show that the differential equation in  $i$  has the form:  $i + RC \frac{di}{dt} = 0$ .
  - c) Verify that  $i = \frac{E}{R} e^{-\frac{t}{RC}}$  is the solution of this differential equation.
- 3) Trace the shape of the curve representing the variation of  $i$  as a function of time.
- 4) Give, in terms of  $R$  and  $C$ , the duration at the end of which the capacitor is practically completely discharged.

#### C – The lightning

In a cloud, the collisions between the water particles give rise to positive and negative charges: The lower part of the cloud becomes negatively charged while its upper part positively charged.

Simultaneously, the ground is charged positively by induction. A capacitor of capacitance  $C = 10^{-10} \text{ F}$  is thus formed having the ground as the positive armature, the lower part of the cloud as the negative armature and the air between them being the insulator. The voltage across its armatures is  $E = 10^8 \text{ V}$ .

In certain conditions, the air between the armatures becomes a conductor of resistance  $R = 5000 \text{ } \Omega$ .

We suppose that the lightning corresponds to the complete discharging of this capacitor through air.

- 1) Calculate the duration of the lightning.
- 2) Determine the maximum current due to the lightning.

### Third exercise (6 points)

#### Nuclear reactions and dating

##### Given:

$$m(\alpha) = 4.00150 \text{ u}; \quad m({}_0^1\text{n}) = 1.00866 \text{ u}; \quad m({}_1^1\text{p}) = 1.00728 \text{ u}; \quad m({}_7^{14}\text{N}) = 13.99924 \text{ u}; \\ m({}_6^{14}\text{C}) = 13.99995 \text{ u}; \quad m({}_8^{17}\text{O}) = 16.99473 \text{ u}; \quad 1 \text{ u} = 931.5 \text{ MeV}/c^2.$$

##### A – Artificial reaction

The first provoked artificial reaction was performed in 1919 by Ernest Rutherford at Cambridge. He bombarded nitrogen nuclei ( ${}_{7}^{14}\text{N}$ ) with  $\alpha$  particles ( ${}_{2}^{4}\text{He}$ ) having great kinetic energies. Oxygen nuclei ( ${}_{8}^{17}\text{O}$ ) and protons ( ${}_{1}^{1}\text{p}$ ) are obtained. The equation corresponding to the reaction relative to one nitrogen nucleus is written as:  ${}_{2}^{4}\text{He} + {}_{7}^{14}\text{N} \longrightarrow {}_{8}^{17}\text{O} + x {}_{1}^{1}\text{p}$

- 1) Show that, specifying the used law,  $x = 1$ .
- 2) a) Calculate the “mass before” and the “mass after” in this nuclear reaction.  
b) Deduce that this reaction needs an amount of energy to be performed.
- 3) We neglect the kinetic energy of the proton and those of nitrogen and oxygen nuclei. Show that, by applying the principle of conservation of the total energy, the kinetic energy of the  $\alpha$  particle is equal to 1.183 MeV.

##### B – Natural reaction

A provoked reaction of nitrogen 14 occurs naturally. Indeed, when, in the upper atmosphere, a neutron of the cosmic radiation hits a nitrogen nucleus ( ${}_{7}^{14}\text{N}$ ), a reaction takes place and produces a carbon nucleus ( ${}_{6}^{14}\text{C}$ ), that is a radioactive isotope of the stable carbon nucleus ( ${}_{6}^{12}\text{C}$ ). The equation corresponding to this reaction is written as:  ${}_0^1\text{n} + {}_{7}^{14}\text{N} \longrightarrow {}_{6}^{14}\text{C} + {}_{1}^{1}\text{p}$

- 1) Calculate the “mass before” and the “mass after” in this nuclear reaction.
- 2) Deduce that this reaction liberates energy.

##### C – Carbon dating

The plants absorb carbon dioxide from the atmosphere formed of both carbon 14 and carbon 12.

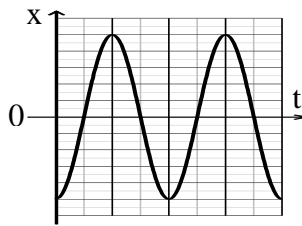
The ratio of these two isotopes is the same in plants and in atmosphere.

When the plant dies, it stops absorbing carbon dioxide. The carbon 14 existing in this plant disintegrates then without being compensated. The period (half-life) of carbon 14 is  $T = 5730$  years.

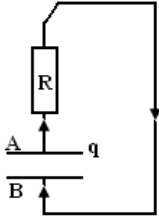
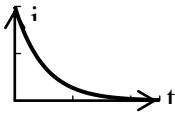
- 1) Calculate, in  $\text{year}^{-1}$ , the radioactive constant  $\lambda$  of carbon 14.
- 2) An analysis of a sample of wood (dead plant), found in an Egyptian tomb, shows that its activity is 750 disintegrations per minute whereas the activity of a plant of the same nature and of the same mass freshly cut is 1320 disintegrations per minute.  
Determine the age of the sample of wood found in the Egyptian tomb.

الدورة العادية للعام 2011	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ساعتان	مشروع معيار التصحيح

### First exercise (7 points)

Part of the Q	Answer	Mark
A1.a	$ME = \frac{1}{2} kx^2 + \frac{1}{2} mv^2 + 0 = \text{constant}$ $\Rightarrow \frac{dM.E}{dt} = 0 \Rightarrow kxv + mvx'' = 0$ $\Rightarrow x'' + \frac{k}{m} x = 0.$	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$
A1.b.i	$\omega_0^2 = \frac{k}{m} \Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$ $T_0 = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{m}{k}}$	$\frac{1}{4}$ $\frac{1}{4}$
A1.b.ii	$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{20}{0.5}} = 2\pi = 6.32 \text{ rd/s}$ $T_0 = 2\pi \sqrt{\frac{0.5}{20}} = 1 \text{ s.}$	$\frac{1}{2}$ $\frac{1}{2}$
A.2	$x(t=0) = x_0 = X_m \cos(\varphi) = -0.1 < 0$ and $v(t=0) = v_0 = -X_m \omega_0 \sin(\varphi) = 0 \Rightarrow \sin(\varphi) = 0 \Rightarrow \varphi = 0 \text{ or } \pi \text{ rad ;}$ if $\varphi = 0, x_0 = X_m > 0 \Rightarrow \varphi = 0 \text{ rd rejected}$ if $\varphi = \pi \text{ rd} \Rightarrow x_0 = -X_m = -0.1 \text{ m, } \varphi = \pi \text{ is accepted}$ $X_m \cos(\varphi) = -0.1 \Rightarrow X_m = 0.1 \text{ m.}$	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$
A.3.a	$v(t) = -X_m \omega_0 \sin(\omega_0 t + \varphi) = -0.2 \pi \sin(2\pi t + \pi) = 0.2 \pi \sin(2\pi t)$	$\frac{1}{4}$
A.3.b	$v_m = 0.2 \pi \text{ m/s} = 0.632 \text{ m/s}$	$\frac{1}{4}$
A.4		$\frac{1}{2}$
A.5.a	$ME = \frac{1}{2} k(x_0)^2 = \frac{1}{2} (20)(0.1)^2 = 0.1 \text{ J.}$	$\frac{1}{2}$
A.5.b	$ME = \frac{1}{2} m(v_m)^2 = 0.1 \Rightarrow v_m = 0.632 \text{ m/s.}$	$\frac{1}{2}$
B.1	The curve (b) represents KE since $V_0 = 0$ and The curve (a) represents the elastic potential energy.	$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$
B.2	$T = 0.5 \text{ s} \Rightarrow T = \frac{T_0}{2}.$	$\frac{1}{4}$ $\frac{1}{4}$

**Second exercise (7 points)**

Part of the Q	Answer	Mark
A.1	In the position (1).	1/4
A.2	At the end of charging, $t \rightarrow \infty$ , $\Rightarrow u_C \rightarrow E(1 - e^{-\infty}) \rightarrow E(1 - 0) \rightarrow E$ . or $t=5RC$ , $e^{-5}=0.007$ , $\Rightarrow u_C=0.993E \approx E$	1/4 1/4
B.1	We turn the switch to the position (2).	1/4
B.2.a		1/2
B.2.b.i	$i = -\frac{dq}{dt}$ with $i > 0$ ; $q$ (decreasing) $\Rightarrow \frac{dq}{dt} < 0 \Rightarrow i > 0$ .	3/4
B.2.b.ii	$u_C = Ri = \frac{q}{C} \Rightarrow \frac{1}{C} \frac{dq}{dt} = R \frac{di}{dt}$ , $i = -\frac{dq}{dt} \Rightarrow -\frac{i}{C} = R \frac{di}{dt}$ $\Rightarrow i + RC \frac{di}{dt} = 0$ .	1
B.2.c	$i = \frac{E}{R} e^{-\frac{t}{RC}} \Rightarrow \frac{di}{dt} = -\frac{E}{R^2 C} e^{-\frac{t}{RC}}$ $i + RC \frac{di}{dt} = \frac{E}{R} e^{-\frac{t}{RC}} + RC \left( -\frac{E}{R^2 C} e^{-\frac{t}{RC}} \right) = \frac{E}{R} e^{-\frac{t}{RC}} - \frac{E}{R} e^{-\frac{t}{RC}} = 0 \checkmark$	1 1/4
B.3		1/2
B.4	$t_{\text{discharge}} = 5RC$	1/2
C.1	$t = 5RC = 5(5000)(10^{-10}) = 25 \times 10^{-7} \text{s}$	1/2
C-2	At $t = 0$ , $i = \frac{E}{R} e^0 = \frac{E}{R}$ $I_{\text{max}} = \frac{E}{R} = \frac{10^8}{5000} = 20000 \text{ A}$ .	1/4 1/4 1/2

### Third exercise (6 points)

Part of the Q	Answer	Mark
A.1	${}^4_2\text{He} + {}^{14}_7\text{N} \longrightarrow {}^{17}_8\text{O} + x {}^1_1\text{H}$ <p>Conservation of charge number: <math>2 + 7 = 8 + x \Rightarrow x = 1</math>;            Or Conservation of mass number: <math>4 + 14 = x + 17 \Rightarrow x = 1</math>;</p>	1/2
A.2.a	$m_{\text{before}} = m({}^4_2\text{He}) + m({}^{14}_7\text{N}) = 4.00150 + 13.99924 = 18.00074 \text{ u.}$ $m_{\text{after}} = m({}^{17}_8\text{O}) + m({}^1_1\text{H}) = 16.99473 + 1.00728 = 18.00201 \text{ u.}$	1/4 1/4 1/4 1/4
A.2.b	This reaction that absorbs energy, because $m_{\text{before}} < m_{\text{after}}$	1/2
A.3	$\cancel{K.E_{(\alpha)}} + m_{(\alpha)}c^2 + \cancel{K.E_{(N)}} + m_{(N)}c^2 = \cancel{K.E_{(O)}} + m_{(O)}c^2 + \cancel{K.E_{(p)}} + m_{(p)}c^2$ $\Rightarrow \Delta m \times c^2 = -K.E_{(\alpha)} ; \Delta m = -0.00127 \text{ u}$ $\Rightarrow K.E_{(\alpha)} = 0.00127 \times \frac{931.5}{\cancel{c^2}} \times \cancel{c^2} = 1.183 \text{ MeV.}$	1/2 1/4 1/4
B.1	$m_{\text{before}} = m({}^1_0\text{n}) + m({}^{14}_7\text{N}) = [1.00866 + 13.99924] = 15.00790 \text{ u.}$ $m_{\text{after}} = m({}^{14}_6\text{C}) + m({}^1_1\text{H}) = [13.99995 + 1.00728] = 15.00723 \text{ u.}$	1/4 1/4
B.2	$m_{\text{before}} > m_{\text{after}}$	1/2
C.1	<p>The radioactive constant: <math>\lambda = \ln 2 / T</math>  <math>= \ln 2 / (5730) = 1.21 \times 10^{-4} \text{ y}^{-1}</math>.</p>	1/4 1/2
C.2	<p>We have <math>A = A_0 e^{-\lambda t} \Rightarrow \ln(A/A_0) = -\lambda t = -0.565</math>  <math>\Rightarrow</math> The age of this piece of wood is: <math>t = 4673 \text{ years.}</math></p>	1/2 1/4 1/2