This exam is formed of three exercises in three pages numbered from 1 to 3
The use of non-programmable calculators is recommended.

First Exercise (7 points)

Horizontal elastic pendulum

The free end of a spring of horizontal axis $(O, \vec{i})$, of negligible mass and of stiffness k = 15 N/m, is connected to a solid (S) of mass m. (S) is free to move on a horizontal table and G, center of mass of (S), may move along the horizontal axis $(O, \vec{i})$.
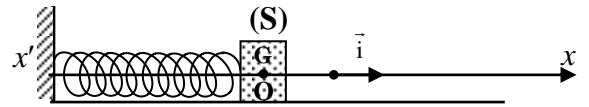
The horizontal plane through G is taken as a gravitational potential energy reference.

A – Theoretical study

G , shifted from its equilibrium position O by a distance x_0 in the positive direction along the axis $(O, \vec{i})$, is released from rest at the instant $t_0 = 0$. (S) thus performs simple harmonic oscillations of proper period T_0.

At an instant t, the abscissa of G is x and the algebraic value of its velocity is $v = \dfrac{dx}{dt}$.

1) Give the expression of the mechanical energy of the system [(S),spring, Earth] at the instant t in terms of k, m, x and v .

2) Derive the second order differential equation in x that governs the motion of G.

3) **a)** The solution of this equation has the form: $x = X_m \cos (\dfrac{2\pi}{T_0} t + \varphi)$. Determine, in terms of the given constants, the expressions of T_0 and X_m and calculate the value of φ.

b) Write down the instantaneous expression of v. Deduce the relation between x_0 , T_0 and the maximum value V_m of v.

B – Experimental study

I – We record, as a function of time, the variations of the abscissa x of G (figure 1) and that of v (figure 2).

Fig. 1   **Fig. 2**
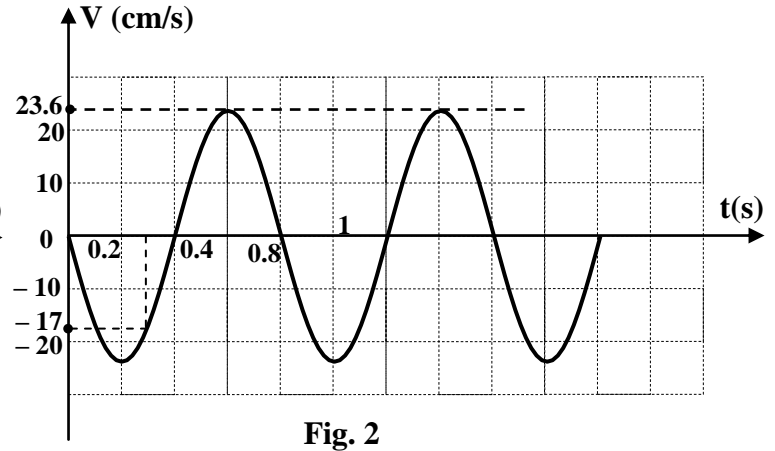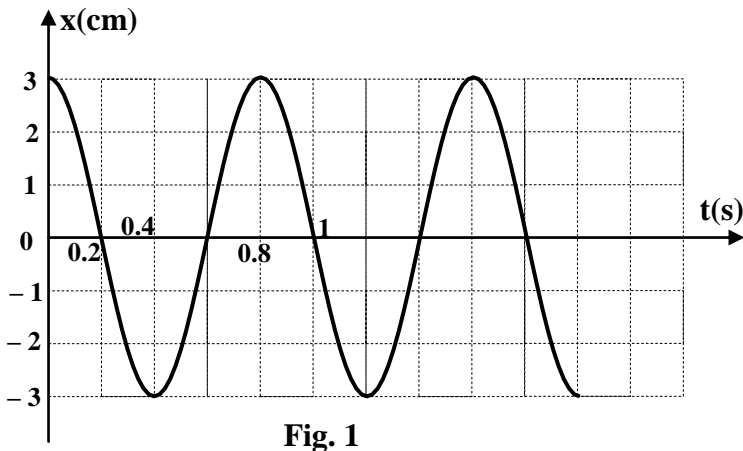
1) Referring to the graphs of figures 1 and 2, specify the value of T_0, that of V_m and the values of x and v at the instant $t_0 = 0$.

2) Determine the mass m of (S).

II – 1) Copy and complete the table below , where K.E is the kinetic energy of (S) , $P.E_e$ is the elastic potential energy of the spring and M.E is the mechanical energy of the system [(S), spring, Earth]

t(s)	0	0.2	0.3
v(m/s)		– 0.236	– 0.17
K.E (J)		6.77×10^{-3}	
x(m)	0.030		– 0.021
$P.E_e$ (J)	6.75×10^{-3}		
M.E (J)			

2) Deduce from the table an indicator that confirms that the oscillations are simple harmonic.

Second Exercise (7 points)
Measurement of the speed of a plane

The aim of this exercise is to measure the speed of a plane using the phenomenon of electromagnetic induction.

A – Motion of a conductor in a uniform magnetic field

A homogeneous metallic rod MN of length ℓ, slides on two horizontal and parallel metallic rails AA' and EE' at a constant velocity $\vec{v}$. During its sliding, the rod remains perpendicular to the rails and its center of mass G moves along the axis Ox.

At the instant $t_0 = 0$, G is at O, the origin of abscissa. At an instant t, the abscissa of G is $x = \overline{OG}$ and $v = \dfrac{dx}{dt}$ is the algebraic value of
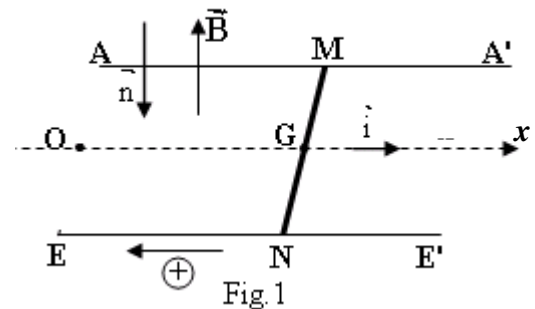
its velocity. The whole set-up formed of the rod and the rails is put within a uniform magnetic field $\vec{B}$ perpendicular to the plane of the horizontal rails (Figure 1).

Fig. 1

1) Determine, at the instant t, the expression of the magnetic flux crossing the surface AMNE in terms of B, ℓ and x, taking into consideration the chosen arbitrary positive direction on figure 1.

2) Explain the existence of an induced e.m.f e across the ends M and N of the rod.

3) Determine the expression of the induced e.m.f e in terms of B, ℓ and v.

4) No current would pass in the rod. Why?

5) Deduce the polarity of the points M and N of the rod and give the expression of the voltage u_{NM} in terms of e.

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B – Measurement of the speed of a plane

A plane is flying horizontally along a straight path with a constant velocity $\vec{v_1}$ of magnitude v_1 within the uniform magnetic field $\vec{B}$ of the Earth.

The vector $\vec{B}$, in the region of flight, has a horizontal component of magnitude $B_h = 2.3 \times 10^{-5}$ T and a vertical component of magnitude $B_v = 4 \times 10^{-5}$ T.

Fig.2
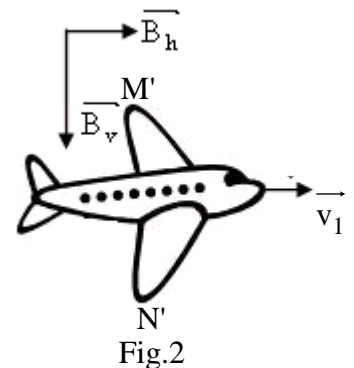
The wings of the plane, considered as a straight and horizontal conductor of length ℓ' = M'N' = 30 m, sweep with time a surface area (Figure 2).

1) a) The magnetic flux of $\overrightarrow{B_h}$ through the swept surface area is zero. Why?
   b) Give the expression of the induced e.m.f e_1 that appears between the ends M' and N' of the wings in terms of B_v, ℓ' and v_1.
2) Determine v_1, if the potential difference across the wings has a value 0.36 V.

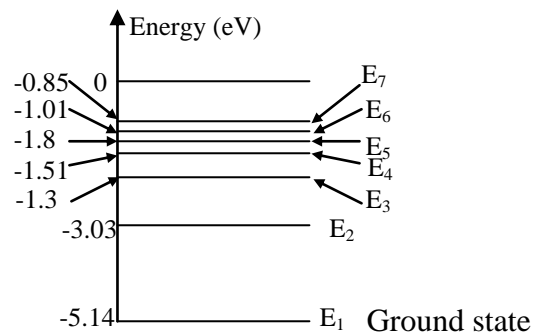## Third Exercise (6 points)
Sodium vapor lamp

Sodium vapor lamps are used to illuminate roads. These lamps contain sodium vapor under very low pressure. This vapor is excited by a beam of electrons that cross the tube containing the vapor. The electrons yield energy to the sodium atoms which give back this received energy during their downward transition towards the ground state in the form of electromagnetic radiations.

Given: h = 6.62×l0^{-34} J.s; c = 3×10^8 ms^{-1}; e = 1.60×10^{-19} C ; 1 nm = 10^{-9}m.

1) What do each of the quantities h, c and e represent?
2) The analysis of the emission spectrum of a sodium vapor lamp shows the presence of lines of well-determined wavelengths λ . The figure below represents some of the lines of this spectrum.

| 330.3 nm | 568.8 nm | 615.4 nm | 819.5 nm | 1138.2 nm |

Doublet 589.0 and 589.6 nm
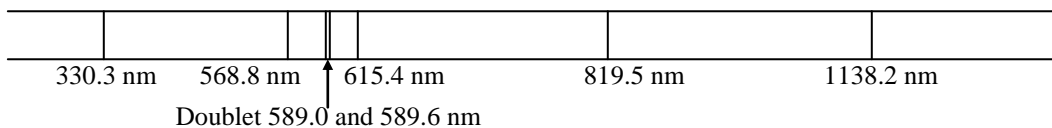
   a) The yellow doublet of wavelengths, in vacuum, $λ_1$= 589.0 nm and $λ_2$ = 589.6 nm is more intense than the other lines.
      i) To what range: visible, infrared or ultraviolet, does each of the other lines of the spectrum belong?
      ii) The sodium vapor lamps are characterized by the emission of yellow light. Why?
   b) Is the visible light emitted by the sodium lamp monochromatic or polychromatic? Justify your answer.
3) a) Referring to the diagram of the energy levels of the sodium atom in the adjacent figure:
      i) Specify an indicator that justifies the discontinuity of the emission spectrum of the sodium vapor lamp.
      ii) Verify that the emission of the line of wavelength $λ_1$ corresponds to the downward transition from the energy level E_2 to the ground state.

Energy (eV)
-0.85 0 ——— E$_7$
-1.01 ——— E$_6$
-1.8 ——— E$_5$
-1.51 ——— E$_4$
-1.3 ——— E$_3$
-3.03 ——— E$_2$
-5.14 ——— E$_1$ Ground state

   b) In fact, the energy level E_2 is double, i.e., it is constituted of two energy levels that are very close to each other. Draw a diagram that shows the preceding downward transition as well as the downward transition corresponding to the emission of the radiation of wavelength $λ_2$.
4) The sodium atom, being in the ground state, is hit successively by the electrons (a) and (b) of respective kinetic energies 1.01 eV and 3.03 eV .
   a) Determine the electron that can interact with the sodium atom.
   b) Specify the state of the sodium atom after each impact.
   c) Deduce, after impact, the kinetic energy of the electron that interacts with the sodium atom.

3

First Exercise (7 points)

A –

1) Expression of the mechanical energy: $ME = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ (¼)

2) There is a conservation of the mechanical energy: $ME = $ constant, then $\dfrac{dE_m}{dt} = 0$

$\frac{1}{2}m\,2\,vV' + \frac{1}{2}k2xx' = 0$. $m\dot{x}\left(\ddot{x} + \dfrac{k}{m}x \right) = 0$; as $\dot{x} \neq 0$ $\forall$ t, then: $\ddot{x} + \dfrac{k}{m}x = 0$ (¾)

3) **a)** $x = X_m \cos\left(\dfrac{2\pi}{T_0}t + \varphi \right)$; $\dfrac{dx}{dt} = \dot{x} = -\dfrac{2\pi}{T_0}X_m \sin\left(\dfrac{2\pi}{T_0}t + \varphi \right)$;

$\dfrac{d^2x}{dt^2} = \ddot{x} = -X_m\left(\dfrac{2\pi}{T_0} \right)^2 \cos\left(\dfrac{2\pi}{T_0}t + \varphi \right)$ By replacing in the differential equation and by

we obtain:

$$\ddot{x} + \left(\dfrac{2\pi}{T_0} \right)^2 x = 0 \quad , \text{ while comparing : } \left(\dfrac{2\pi}{T_0} \right)^2 = \dfrac{k}{m}$$

$$\Rightarrow T_0 = 2\pi\sqrt{\dfrac{m}{k}}$$

For $t_0 = 0$, $X_m \cos(\varphi) = x_0 > 0$ and $v = -\dfrac{2\pi}{T_0}X_m \Rightarrow \sin(\varphi) = 0$

$\Rightarrow \varphi = 0$ or π. But $X_m > 0 \Rightarrow \cos\varphi = 1$ thus $X_m = x_0$ and $\varphi = 0$ (1¾)

b) $v = \dfrac{dx}{dt} = \dot{x} = -\dfrac{2\pi}{T_0}x_0 \sin\left(\dfrac{2\pi}{T_0}t \right)$; $\Rightarrow |V_m| = \dfrac{2\pi}{T_0}x_0$ (1)

B – I – 1) $T_0 = 0.8$ s, $x_0 = 3$ cm and $V_m = 23.6$ cm/s or 0.236 m/s

At $t_0 = 0$, $x_0 = 3$ cm and $v_0 = 0$.   (1)

2) $T_0 = 2\pi\sqrt{\dfrac{m}{k}} \Rightarrow m = \left(\dfrac{T_0}{2\pi} \right)^2 k = \left(\dfrac{0.8}{2\pi} \right)^2 \times 15 = 0.243$ kg.   (¾)

II – 1) (1¼)

t(s)	0	0.2	0.3
v(m/s)	0	- 0.236	-0.17
KE (J)	0	6.77×10^{-3}	3.51×10^{-3}
x(m)	0.030	0	-0.021
EPE (J)	6.75×10^{-3}	0	3.31×10^{-3}
ME (J)	6.75×10^{-3}	6.77×10^{-3}	6.82×10^{-3}

2) the mechanical energy is approximately the same $\Rightarrow$ it is constant.   (¼)

Second Exercise (7 points)

A –

1) $\varphi = \vec{B}\,\vec{n}\,S\cos\alpha = -BS = -B\ell x$.   (¾)

2) The magnetic flux varies, therefore an induced emf e appears across the extremities N and M of the rod   (½)

3) $e = -\dfrac{d\varphi}{dt} = B\ell\dfrac{dx}{dt} = B\ell v$.   (¾)

4) The circuit is open $\Rightarrow$ $i = 0$.   (½)

5) $u_{NM} = e - ri$ $(i = 0) \Rightarrow u_{NM} = e > 0$,
The point N is positive and the point M is negative.
$U_{NM} = e = B\ell v$ (2)

B –

1) **a)** $\varphi_h = B_h S\cos90^0 = 0$ (½)

b) $\varphi_V = B_V S\cos0^0 = B_V\,\ell' x$
$e = -\dfrac{d\varphi}{dt} = -B_v\ell'\dfrac{dx}{dt} = -B_v\ell'v_1$.   (1)

2) $|u_{NM}| = B_v\ell'v_1 \Rightarrow v_1 = \dfrac{0.36}{4\times10^{-5}\times30} = 300$ m/s.   (1)

Third Exercise (6 points)

1) h: Planck's constant; c: speed of light in vacuum, e: elementary charge   (½)

2) **a) i)** 330.3 nm ultraviolet domain;
      568.8 nm, 589 nm and 615.4 nm visible domain;
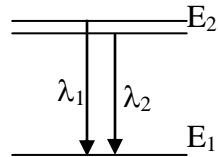   819.5 nm and 1138.2 nm infra-red domain.   (¾)

    ii) Because this yellow light is much more intense than the others   (¼)

  b) It is polychromatic because it is made of several radiations of different frequencies   (¼)

3) **a) i)** The discontinuity of the emission spectrum is justified by the discontinuous energy levels of the sodium atom. (½)

    ii) $E = \dfrac{hc}{\lambda} = \dfrac{6.62\times10^{-34}\times3\times10^{8}}{589} = 3.37\times10^{-19}$ J or $E = \dfrac{3.37\times10^{-19}}{1.6\times10^{-19}} = 2.11$ eV. $E_1 + E = -5.14 + 2.11 = -3.03$ eV   (1¼)

b)   (½)

$$\begin{array}{l} \rule{3cm}{0.5pt}\; E_2 \\[4pt] \lambda_1 \;\; \lambda_2 \\[4pt] \rule{3cm}{0.5pt}\; E_1 \end{array}$$

4) **a)** $1.01 + (-5.14) = -4.13$ eV , this energy level does not exist $\Rightarrow$ the electron (a) does not interact with the atom.
    $3.03 + (-5.14) = -2.11$ eV , $-3.03 < -2.11 < -1.93$ ev , $\Rightarrow$ the electron (b) interacts with the atom.   **(1)**

  b) In case of the electron (a) the atom remains in the ground state.
    In case of the electron (b) the atom attains level E_2.   (½)

c) For the electron (b), K.E $= 3.03 - (-3.03 + 5.14) = 0.92$ eV   (½)