## This exam is formed of three exercises in 3 pages

## The use of non-programmable calculators is recommended

## First exercise (7pts) Mechanical interaction

The object of this exercise is to study some physical quantities of a system whose parts are in mechanical interaction.
For that, we use two pucks (A) and (B), of respective masses $m_{A}=100 \mathrm{~g}$ and $m_{B}=120 \mathrm{~g}$, that may move without friction on a horizontal table. Each puck is surrounded by an elastic steel shock ring of negligible mass. The two pucks are connected by a massless and inextensible taut thread thus compressing the steel shock rings. The system ( S ) thus formed is at rest. (Figure 1)
We burn the thread; the shock rings stretch and the pucks repel each other. The system (S) thus formed of the two pucks and the shock rings is said to "explode".


The positions of the center of mass of each puck are registered at successive instants separated by a constant time interval $\tau=50 \mathrm{~ms}$.
Figure (2) represents, on the axis $x^{\prime} x$, the dot-prints of the positions of the centers of masses $G_{A}$ and $G_{B}$ of the two pucks after the «explosion».


Fig. 2

1) Using the document of figure (2), show that, after explosion:
$a$ - The motion of each puck is uniform;
$\boldsymbol{b}$ - The speeds of $(\mathrm{A})$ and $(\mathrm{B})$ are $\mathrm{V}_{\mathrm{A}}=1.2 \mathrm{~m} / \mathrm{s}$ and $\mathrm{V}_{\mathrm{B}}=1 \mathrm{~m} / \mathrm{s}$ respectively.
2) Verify the conservation of the linear momentum of the system (S) during explosion.
3) Applying Newton's second law $\frac{\mathrm{d} \overrightarrow{\mathrm{P}}}{\mathrm{dt}}=\sum \overrightarrow{\mathrm{F}_{\mathrm{ext}}}$ on each puck and assuming that the time interval of the explosion $\Delta \mathrm{t}=0.05$ s is so small that $\frac{\Delta \overrightarrow{\mathrm{P}}}{\Delta \mathrm{t}}$ has the same value as $\frac{\mathrm{d} \overrightarrow{\mathrm{P}}}{\mathrm{dt}}$,
$\boldsymbol{a}$ - Determine the forces $\overrightarrow{\mathrm{F}}_{\mathrm{A} \rightarrow \mathrm{B}}$ and $\overrightarrow{\mathrm{F}}_{\mathrm{B} \rightarrow \mathrm{A}}$ exerted respectively by (A) on (B) and by (B) on (A).
$b$ - Verify the principle of interaction.
4) The system (S) possesses a certain energy before the explosion.
$\boldsymbol{a}$ - Specify the part of (S) storing this energy.
$\boldsymbol{b}$ - In what form is this energy stored?
$c$ - Determine the value of this energy.

## Second exercise ( $6^{1 / 2} \mathbf{~ p t s}$ ) Charging and discharging of a capacitor

The object of this exercise is to study the functioning of an apparatus that allows the illumination and the putting off automatically of a lamp at the end of an adjustable time interval $\mathrm{t}_{1}$.

## A-Principle of functioning of the apparatus

Consider a source of DC voltage of value E, a push-button switch, a resistor of resistance R and a capacitor of adjustable capacitance C that is initially neutral. We connect up the circuit represented in figure 1.

## 1) Charging of the capacitor

The push-button switch is pushed to position (1).
Then capacitor undergoes charging.
$a$ - The duration of the charging of the capacitor is very short.Why?
$\boldsymbol{b}$ - What would the value of the voltage $\mathrm{u}_{\mathrm{C}}=\mathrm{u}_{\mathrm{AB}}$ across the capacitor thus charged be?


## 2) Discharging of the capacitor

The capacitor being charged, we release the push-button switch that returns automatically to position (2) at the instant $\mathrm{t}_{0}=0$. The capacitor undergoes discharging through the resistor.
$\boldsymbol{a}$ - Determine, at an instant t , the differential equation that governs the variations of $\mathrm{u}_{\mathrm{C}}$ as a function of time.
$\boldsymbol{b}$ - The solution of the previous differential equation has the form $\mathrm{u}_{\mathrm{C}}=\mathrm{a} e^{\frac{-t}{\tau}}$ where a and $\tau$ are positive constants. Determine the expressions of a and $\tau$ in terms of $\mathrm{E}, \mathrm{R}$ and C .
$c$ - Show that, for $t=\tau$, the voltage across the capacitor is equal to $37 \%$ of its value at the instant $t_{0}=0$.

## B - Use of the apparatus

The lighting apparatus is represented by the circuit of figure 2 where $E=10 \mathrm{~V}$ and the resistor is replaced by a lamp of resistance $R=3 \mathrm{k} \Omega$.
The lamp illuminates as long as the voltage across its terminals is greater or equal to a limiting voltage denoted by U .

1) $\boldsymbol{a}$ - Using the solution of the differential equation [given in the question (A-2-b)], determine the expression of the duration $t_{1}$ of illumination of the lamp in terms of $\mathrm{U}, \mathrm{E}$ and $\tau$.
$b$ - Calculate $t_{1}$ for $U=3.7 \mathrm{~V}$ and $\mathrm{C}=2 \times 10^{-2} \mathrm{~F}$.

2) We keep the same lamp and the same DC source. Which component of the apparatus must be modified and how in order to increase the duration of illumination of the lamp?

Cobalt ${ }_{27}^{60} \mathrm{Co}$ is a $\beta^{-}$radioactive. The daughter nucleus ${ }_{Z}^{A} \mathrm{Ni}$ undergoes a downward transition to the ground state. The energy due to this downward transition is $\mathrm{E}(\gamma)=2.5060 \mathrm{MeV}$.
The $\beta^{-}$particle is emitted with a kinetic energy $\operatorname{K.E}\left(\beta^{-}\right)=0.0010 \mathrm{MeV}$.
Numerical data: mass of the ${ }_{27}^{60} \mathrm{Co}$ nucleus: 59.91901 u ;
mass of the ${ }_{\mathrm{Z}}^{\mathrm{A}} \mathrm{Ni}$ nucleus : 59.91544 u ;
mass of an electron : $5.486 \times 10^{-4} \mathrm{u}$;
$1 \mathrm{u}=931.5 \mathrm{MeV} / \mathrm{c}^{2}$;
$1 \mathrm{MeV}=1.6 \times 10^{-13} \mathrm{~J}$.

## A -Study of the disintegration

1) Determine $A$ and $Z$.
2) Calculate, in $u$, the mass defect $\Delta \mathrm{m}$ during this disintegration.
3) Deduce, in MeV , the energy E liberated by this disintegration.
4) During this disintegration, the daughter nucleus is practically obtained at rest.

In what form of energy does E appear?
5) $\boldsymbol{a}$ - Deduce, from what preceded, that the electron, emitted by the considered disintegration, is accompanied by a certain particle.
$\boldsymbol{b}$ - Give the name of this particle.
$c$ - Give the charge number and the mass number of this particle.
$d$ - Deduce in MeV , the energy of this particle.
6) Write down the global equation of this disintegration.

## B - The use of cobalt 60

In medicine, we use a source of radioactive cobalt ${ }_{27}^{60} \mathrm{Co}$ of activity $\mathrm{A}=6 \times 10^{19} \mathrm{~Bq}$.
The emitted $\beta^{-}$particles are absorbed by the living organism.

1) The energy of the particle mentioned in the question (A-5) is not absorbed by the living organism. Why?
2) Calculate, in watt, the power transferred to the organism.
3) This large power is used in radiotherapy. What is its effect?

## Solution

## First exercise (7 pts)

1) 

a- The distances covered during equal time intervals are equal.
b- $\quad V_{B}=\frac{d}{t}=\frac{d}{4 \tau}=\frac{0.2}{4 \times 0.05}=1 \mathrm{~m} / \mathrm{s}$.
$\mathrm{V}_{\mathrm{A}}=\frac{0.24}{4 \times 0.05}=1.2 \mathrm{~m} / \mathrm{s}$.
( $3 / 4 \mathbf{~ p t}$ )
( $3 / 4 \mathrm{pt}$ )
2) $\overrightarrow{P_{\text {before }}}=\overrightarrow{0} ; \overrightarrow{\mathrm{P}_{\text {after }}}=m_{A} \overrightarrow{V_{A}}+m_{B} \overrightarrow{V_{B}}$

$$
=0.1(-1.2 \dot{\mathrm{i}})+0.12(\overrightarrow{\mathrm{i})}=\overrightarrow{0}
$$

$\Rightarrow \overrightarrow{\mathrm{P}_{\text {before }}}=\overrightarrow{\mathrm{P}_{\text {atter }}}$;
The linear momentum is conserved for the system formed of the two pucks.
3)
a- Newton $2^{\text {nd }}$ Law applied on A gives:

$$
\begin{aligned}
\frac{\overrightarrow{\mathrm{dP}}}{\mathrm{dt}} & =\overrightarrow{\mathrm{m}_{\mathrm{A}} \mathrm{~g}}+\overrightarrow{\mathrm{N}_{\mathrm{A}}}+\overrightarrow{\mathrm{F}_{\mathrm{B} \rightarrow \mathrm{~A}}}=\overrightarrow{\mathrm{F}_{\mathrm{B} \rightarrow \mathrm{~A}}} \\
& =\frac{0.1(-1.2-0) \dot{\mathrm{i}}}{0.05}=-2.4 \dot{i} \cdot(\mathbf{1} \mathbf{~ p t}) \\
\frac{\overrightarrow{\mathrm{dP}}}{\mathrm{dt}} & =\overrightarrow{\mathrm{m}_{\mathrm{B}} \mathrm{~g}}+\overrightarrow{\mathrm{N}_{\mathrm{B}}}+\overrightarrow{\mathrm{F}_{\mathrm{A} \rightarrow \mathrm{~B}}}=\overrightarrow{\mathrm{F}_{\mathrm{A} \rightarrow \mathrm{~B}}} \\
& =\frac{0.12(1-0) \dot{\mathrm{i}}}{0.05}=2.4 \dot{i} .(\mathbf{1} \mathbf{~ p t})
\end{aligned}
$$

b- $\overrightarrow{\mathrm{F}_{\mathrm{B} \rightarrow \mathrm{A}}}=-\overrightarrow{\mathrm{F}_{\mathrm{A} \rightarrow \mathrm{B}}} \quad(1 / 2 \mathbf{p t})$
4)
a- The deformed elastic shock ring.
b- Elastic potential energy.
c- The mechanical energy of the system is conserved because the system is isolated ( The system does not exchange energy with the surroundings) ; (Elastic potential energy is transformed into kinetic energy):
$\mathrm{M} \cdot \mathrm{E}=\mathrm{K} \cdot \mathrm{E}+\mathrm{P} \cdot \mathrm{E}_{\mathrm{el}}=\mathrm{M} \cdot \mathrm{E}_{\text {before }}=\mathrm{M} \cdot \mathrm{E}_{\text {after }}=0+\mathrm{P} \cdot \mathrm{E}_{\mathrm{el}}=\mathrm{K} \cdot \mathrm{E}+0$
$\Rightarrow$ P. $\mathrm{E}_{\text {el. }}=\frac{1}{2} \mathrm{~m}_{\mathrm{A}} \mathrm{V}_{\mathrm{A}}^{2}+\frac{1}{2} \mathrm{~m}_{\mathrm{B}} \mathrm{V}_{\mathrm{B}}^{2}=0.132 \mathrm{~J} \quad(\mathbf{1} \mathbf{p t})$

## Second exercise ( $6^{1 / 2} \mathrm{pts}$ )

A-

1) a- Because $\tau=\mathrm{RC} \approx 0$ during charging. ( $1 / 2 \mathbf{p t}$ )

$$
\mathbf{b}-u_{C}=\mathrm{E} \quad(1 / 4 \mathbf{p t})
$$

2) $\mathbf{a}-\mathrm{u}_{\mathrm{C}}=\mathrm{Ri}$ and $\mathrm{i}=-\mathrm{C} \frac{\mathrm{du}}{\mathrm{C}}$

$$
\Rightarrow \mathrm{u}_{\mathrm{C}}+\mathrm{RC} \frac{\mathrm{du}_{\mathrm{C}}}{\mathrm{dt}}=0 \quad\left(\mathbf{1}^{11 / 4} \mathbf{p t}\right)
$$

b- $\frac{\mathrm{du}_{\mathrm{C}}}{\mathrm{dt}}=-\frac{\mathrm{a}}{\tau} \mathrm{e}^{-\frac{\mathrm{t}}{\tau}} \Rightarrow$ a $e^{\frac{-t}{\tau}}+\operatorname{RC}\left(-\frac{\mathrm{a}}{\tau} \mathrm{e}^{-\frac{\mathrm{t}}{\tau}}\right)=0$

$$
\Rightarrow 1-\frac{\mathrm{RC}}{\tau}=0 \Rightarrow \tau=\mathrm{RC} \text {; }
$$

For $\mathrm{t}=0, \mathrm{u}_{\mathrm{C}}=\mathrm{E}=\mathrm{a} . \quad\left(\mathbf{1}^{1 / 2} \mathbf{p t}\right)$
c- If $t=\tau, u_{C}=\mathrm{Ee}^{-1}=0.37 \mathrm{E}=37 \% \mathrm{E} .(\mathbf{1} \mathbf{p t})$

B-

1) $\mathbf{a}-u_{C}=E e^{\frac{-t}{\tau}} \Rightarrow U=E e^{-\frac{t_{1}}{R C}}$

$$
\Rightarrow \frac{\mathrm{t}_{1}}{\mathrm{RC}}=\ln \frac{\mathrm{E}}{\mathrm{U}}
$$

$$
\Rightarrow \mathrm{t}_{1}=\mathrm{RC} \ln \frac{\mathrm{E}}{\mathrm{U}}=\tau \ln \frac{\mathrm{E}}{\mathrm{U}} \cdot(\mathbf{1} \mathbf{p t})
$$

$$
\mathbf{b}-\mathrm{t}_{1}=60 \mathrm{~s} . \quad(1 / 2 \mathbf{p t})
$$

2) Capacitor $(1 / 4 \mathbf{p t})$

We must increase the value of $C$, Because $t_{1}$ is proportional to C. $(1 / 4 \mathbf{p t})$

## Third exercise ( $6^{1 / 2} \mathrm{pts}$ )

A -

1) The conservation of charge number and of mass number gives :
$\mathrm{Z}=28$ and $\mathrm{A}=60 \quad(1 / 2 \mathbf{p t})$
2) $\Delta \mathrm{m}=\mathrm{m}_{\text {before }}-\mathrm{m}_{\text {after }}$
$=59.91901-(59.91544+0.0005486)$
$=0.0030214 \mathrm{u} . \quad(1 / 2 \mathbf{p t})$
3) $\Delta \mathrm{m}=0.0030214 \times 931.5 \mathrm{MeV} / \mathrm{c}^{2}$

$$
=2.8144 \mathrm{MeV} / \mathrm{c}^{2}
$$

$\mathrm{E}=\Delta \mathrm{m} \times \mathrm{c}^{2}=2.8144 \mathrm{MeV} \quad(3 / 4 \mathbf{p t})$
4) E appears in the form of the kinetic energy of the obtained particles and of radiant energy of the $\gamma$ photon.
5) a-K.E $\left(\beta^{-}\right)+E(\gamma)=0.0010+2.5060$

$$
=2.507 \mathrm{MeV}
$$

Is smaller than $\mathrm{E}=2.8144$; therefore there is no conservation of energy, thus there is a necessity to admit the emission of another particle other tan electron. ( $\mathbf{1} \mathbf{~ p t )}$
b- Antineutrino . ( $1 / 4 \mathbf{p t )}$
$\mathbf{c}-\mathrm{Z}=0$ and $\mathrm{A}=0 .(1 / 2 \mathbf{p t})$
d- $\mathrm{E}=2.8144-2.507=0.3074 \mathrm{MeV} .(1 / 2 \mathbf{p t})$
6) ${ }_{27}^{60} \mathrm{Co} \rightarrow{ }_{28}^{60} \mathrm{Ni}+{ }_{-1}^{0} \mathrm{e}+{ }_{0}^{0} \overline{\mathrm{v}}+\gamma(1 / 4 \mathbf{p t})$

B -

1) Because the antineutrino does not interact with matter.
( $1 / 4 \mathbf{p t}$ )
2) The activity corresponds to $6 \times 10^{19}$ disintegrations per second, that means à $6 \times 10^{19}$ electrons emitted per second
$\Rightarrow \mathrm{P}=6 \times 10^{19} \times 0.0010 \times 1.6 \times 10^{-13} \mathrm{~W}=9.6 \mathrm{~kW}$.
( $\left.1^{1 / 4} \mathbf{~ p t}\right)$
3) The destruction of cells. $(1 / 4 \mathbf{p t})$
