

*This exam is formed of three obligatory exercises
in three pages numbered from 1 to 3.
The use of non-programmable calculators is allowed.*

First exercise (6 pts) Determination of the speed of a bullet

A gun is used to shoot bullets, each of mass $m = 20 \text{ g}$, with a horizontal velocity \vec{V}_0 of value V_0 .

In order to determine V_0 , we consider a setup formed of a wooden block of mass $M = 1 \text{ kg}$, suspended from the ends of two inextensible string of negligible mass and of the same length (figure 1).

This setup can be taken as a block of wood suspended from the free end a string of length $\ell = 1 \text{ m}$, initially at rest in the equilibrium position at G_1 .

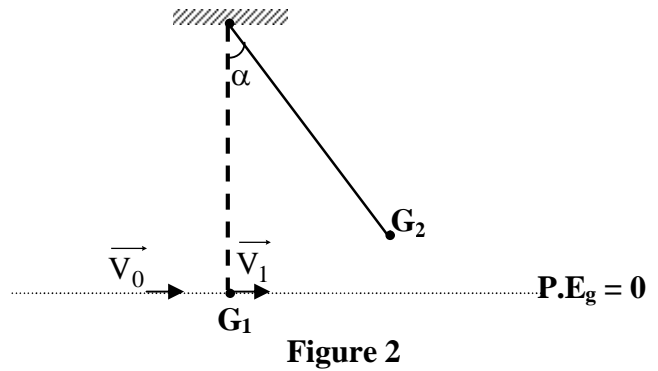
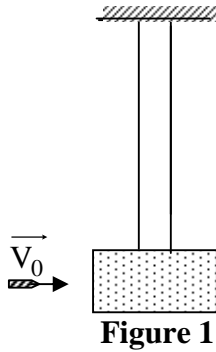
A bullet having the velocity \vec{V}_0 hits the block and is embedded in at the level of the center of mass G of the block.

Just after impact, the system (block, bullet) moves with a horizontal velocity \vec{V}_1 . The pendulum thus attains a maximum angular deviation $\alpha = 37^\circ$.

G_1 and G_2 are the respective positions of G in the equilibrium position and in the highest position.

Take the horizontal plane through G_1 as a gravitational potential energy reference (figure 2).

Neglect friction with air and take $g = 9,8 \text{ m/s}^2$.



1. During a collision, which one of the two physical quantities, the linear momentum or the kinetic energy of the system does not remain always conserved?
2. Determine the expression of the value of V_1 of the velocity \vec{V}_1 in terms of M , m and V_0 .
3. a) Determine, just after impact, the mechanical energy of the system (pendulum, Earth) in terms of V_0 , M , and m .
b) Determine, in terms of M , m , g , ℓ and α , the mechanical energy of the system (pendulum, Earth) at point G_2 .
c) Deduce the value of V_0 .
4. Verify the answer of question (1).

Second exercise (7 pts) Determination of the nature and the characteristics of an electric components

Consider three electric components of different natures. One of these components is a resistor of resistance R , another one a capacitor of capacitance C and the last one of inductance L and of negligible resistance.

A- Nature of each component

In order to determine the nature of each component, we consider a DC generator G , a resistor of resistance r , an ammeter A and a switch K .

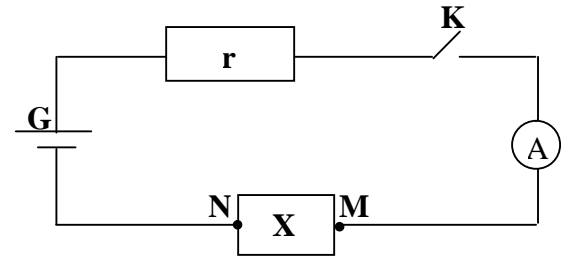


Figure 1

1. First experiment

We connect the circuit represented in figure (1).

We connect, between M and N , one of the components called X , and, then, we close K . The ammeter reads then a certain value which decreases to zero after a certain time.

Determine the nature of the component X .

2. Second experiment

We perform the above experiment again by replacing the component X by the component, called Y . The ammeter reads, in this case, a constant value.

Determine the nature of the component Y .

3. The third component, called Z , is connected alone between M and N . Indicate its nature and specify its effect on the growth of the current in the circuit.

B- Characteristics of the components

1. Value of C

The capacitor of capacitance C is fed by a function generator (LFG) [figure 2] delivering a sinusoidal alternating voltage $u = U \sqrt{2} \sin 2\pi f t$, of effective value $U = 1V$ and adjustable frequency f . Take $0,32 \pi = 1$.

We give f different values, and we measure, using the ammeter the corresponding values of the effective current. I carried by the circuit. The graph of figure (3) represents the variation of I as a function of f .

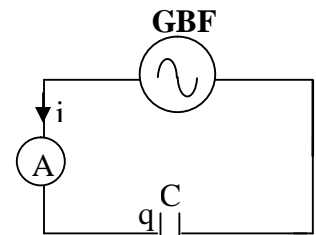


Figure 2

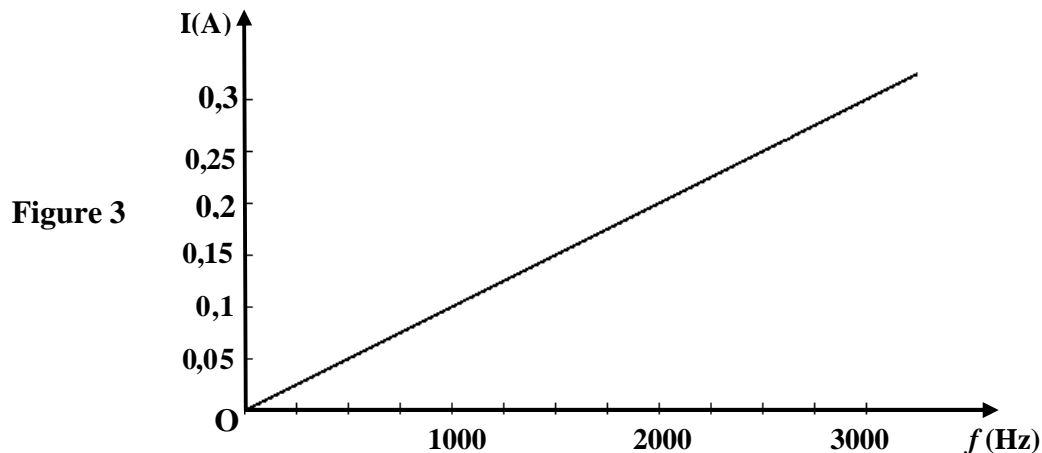


Figure 3

- According to the graph, we can write $I = B \times f$ where B is a constant. Calculate B .
- Using the relation $i = dq/dt$, give the expression of B in terms of U and C .
- Determine the value of C .

2. Value of L

The three components X, Y and Z are now connected in series across the given function generator (LFG). (figure 4).

f is made to vary while keeping the effective value U . We find that the effective current I carried by the circuit attains a maximum value I_0 for $f_0 = 20$ Hz.

- The existence of the maximum value I_0 of I shows evidence of a physical phenomenon. Give the name of this phenomenon.
- Knowing that $C = 1,6 \times 10^{-5}$ F, determine the value of L.

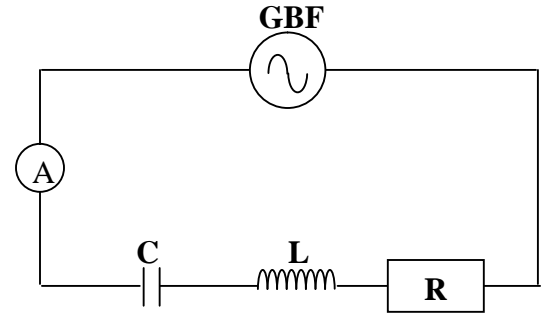


Figure 4

Third exercise (7 pts)

The carbon 14

The object of this exercise is to show evidence of some characteristic properties of the radio element $^{14}_6C$ and to show the procedure followed to know the age of a wooden fossil.

Given :

mass of a proton : $m_p = 1,00728$ u ;

mass of a neutron : $m_n = 1,00866$ u ;

mass of a nucleus $^{14}_6C = 14,0065$ u ;

mass of a nucleus $^{14}_7N = 14,0031$ u ;

1 u = $931,5$ MeV/ c^2 ;

Molar mass of $^{14}_6C = 14$ g \times mol $^{-1}$;

Avogadro's number : $\mathcal{N} = 6,02 \times 10^{23}$ mol $^{-1}$.

A - Formation of carbon 14

In the high atmosphere the carbon isotope $^{14}_7N$ is obtained by the impact of a nitrogen $^{14}_6C$ with a neutron.

- The nuclides $^{12}_6C$ and $^{14}_6C$ are two isotopes. Why?
- Write the equation of the formation of $^{14}_6C$.
- Identify the emitted particle.

B – Disintegration of carbon 14

Carbon 14 is radioactive β^- emitter. It disintegrates to give nitrogen $^{14}_7N$.

- The emission of a β^- particle is due to the disintegration of a nucleon inside the nucleus.
- Calculate the binding energy per nucleon of each of the nuclei $^{14}_6C$ and $^{14}_7N$.
- In fact, a radioactive decay leads to a more stable state. Justify this statement taking into account the preceding results.
- The activity of a substance containing carbon 14 is determined using a counter of β^- particles. A sample of wood containing 0.05g of carbon 14 of radioactive period $T = 5570$ years is exposed to the counter.

Determine:

- The radioactive constant λ of carbon 14.
- The number of carbon 14 nucle contained in this sample at the instant of exposure.
- The activity of the sample at the considered instant.

C- Age of a wood fossil

We intend to determine the age of a piece of wood fossil. We expose this piece to the counter of β^- particles; it indicates 100 disintegration in 5 minutes. Knowing that a piece of the same wood, freshly cut, gives 1000 disintegrations in 5 minutes, determine the age of the wood fossil.

Solution

First exercise (6 pts.)

1. The kinetic energy of the system (bullet, block) (1/4pt.)

2. $\vec{P}_{\text{before}} = \vec{P}_{\text{after}}$ (1/4 pt)

$$m \vec{V}_0 = (M+m) \vec{V}_1 \quad (1/4 \text{ pt}) \quad \text{Thus :} \quad V_1 = \frac{mV_0}{(M+m)} \quad (1/4 \text{ pt.})$$

3. a. M.E = P.E_g + K.E (1/4pt.)

$$\text{M.E} = 0 + \text{K.E} = \frac{1}{2}(M+m)V_1^2 \quad (1/4\text{pt.})$$

$$\text{M.E} = \frac{1}{2}(M+m) \left[\frac{mV_0}{(M+m)} \right]^2 = \frac{1}{2} \frac{m^2 V_0^2}{(M+m)} \quad (1/4 \text{ pt.})$$

b. M.E = (M+m)gh (1/4pt.)

$$h = l - l \cos \alpha = l(1 - \cos \alpha) \quad (1/2\text{pt})$$

$$\text{Thus : M.E} = (M+m)g l(1 - \cos \alpha) \quad (1/4\text{pt.})$$

c. The friction is neglected and the M.E of (pendulum, Earth) is conserved. (1/2pt.)

$$\frac{1}{2} \frac{m^2 V_0^2}{(M+m)} = (M+m)g l(1 - \cos \alpha)$$

$$V_0 = \frac{(M+m)}{m} \sqrt{2gl(1 - \cos \alpha)} \quad (1\text{pt.})$$

$$V_0 = 101,3 \text{ m/s} \quad (1/2 \text{ pt.})$$

4. K.E_{before} = 1/2 m V₀² (1/4pt)

$$\text{K.E}_{\text{before}} = 102,6 \text{ J} \quad (1/4\text{pt})$$

$$\text{K.E}_{\text{after}} = \frac{1}{2}(M+m)V_1^2$$

$$= \frac{1}{2} \frac{m^2 V_0^2}{(M+m)} \quad (1/4\text{pt})$$

$$E_{\text{after}} = 2 \text{ J} \quad (1/4\text{pt})$$

K.E_{before} is ≠ from K.E_{after} (1/4pt)

Second exercise (7 pts.)

A-

1. X is a capacitance because the current decreases till zero. **(3/4 pt.)**

2. Y is a resistance because the value of the current remains constant. **(3/4 pt)**

3. Z is a coil of inductance. It delays the growth of the current. **(3/4 pt)**

B-1.a) $B = 10^{-4} \text{ A / Hz}$ **(1 pt)**

b) We have : $i = \frac{dq}{dt} = \frac{Cdu_c}{dt}$

$i = C U \sqrt{2} 2\pi f \cos 2\pi ft$.Or $i = I \sqrt{2} \cos 2\pi ft \Rightarrow$
 $I = 2\pi C U f = B f \Rightarrow B = 2\pi C U$ **(13/4pt.)**

c) $C = B / 2\pi U = 10^{-4} / 2\pi = 16 \times 10^{-6} \text{ F}$ **(1/2 pt)**

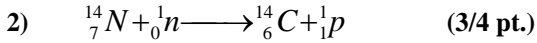
2.a) Current resonance phenomenon. **(1/2 pt)**

b) $f_0 = \frac{1}{2\pi\sqrt{LC}}$ **(1/2 pt)**

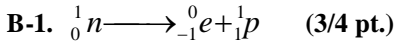
$\Rightarrow L = 0,11 \text{ H.}$ **(1/2 pt)**

Third exercise (7 pts)

A-1) The nuclides have the same charge number Z and the numbers of mass A is different. **(1/2 pt.)**



3) The emitted particle is a proton (or hydrogen nucleus) **(1/4pt.)**



2. The binding energy of the nucleus of mass m_x is : $E_b = \Delta m \cdot c^2$ **(1/4pt.)**

with $\Delta m = [Zm_p + (A-Z)m_n] - m_x$ **(1/4pt)**

The binding energy per nucleon is $\frac{E_b}{A}$. **(1/4pt)**

- For the nucleus ${}^{14}_6C$ we have :

$\Delta m = 6 \times 1,00728 + 8 \times 1,00866 - 14,0065$

$\Delta m = 0,10646 \text{ u} ; E_b = 99,16749 \text{ MeV}$

$\frac{E_b}{A} = 7,083 \text{ MeV}$ **(1/2 pt)**

- For the nucleus ${}^{14}_7N$ we have ;

$\Delta m = 7 \times 1,00728 + 7 \times 1,00866 - 14,0031$

$\Delta m = 0,10848 \text{ u} ; E_b = 101,04912 \text{ MeV}$

$\frac{E_b}{A} = 7,217 \text{ MeV}$ **(1/2 pt)**

3. The nucleus ${}^{14}_7N$, has a binding energy per nucleon more than that of ${}^{14}_6C$; the nucleus ${}^{14}_7N$ is more stable from the nucleus ${}^{14}_6C$.

(1/4pt)

4.a) $\lambda = \frac{0,693}{T}$; **(1/4pt)**

$\lambda = 1,244 \times 10^{-4} \text{ year}^{-1} = 3,94 \times 10^{-12} \text{ s}^{-1}$ **(1/4pt)**

b) $n = \frac{0,05 \times 6,02 \times 10^{23}}{14} = 215 \times 10^{19} \text{ nuclei}$ **(1/2pt)**

c) $A = \lambda \times n$ **(1/4 pt)** ; $A = 8471 \times 10^{10} \text{ Bq}$. **(1/4pt)**

C- $A_0 = 200 \text{ des./mn}$ $A = 20 \text{ des./mn}$

$A = A_0 e^{-\lambda t}$ **(1/4pt)** ; $t = \frac{\ln \frac{A_0}{A}}{\lambda} = 18509 \text{ years}$ **(1pt)**