

الاسم:  
الرقم:

مسابقة في مادة الفيزياء  
المدة: ساعتان ونصف

**This exam is formed of four obligatory exercises in four pages.**  
**The use of non-programmable calculator is recommended.**

### Exercise 1 (5 pts)

#### Motion on a slide

In a park, a child plays on a slide.

The child, considered as a particle, has a mass  $M = 20$  kg.

He climbs to point A the top of the slide, and then slides down without initial velocity to point B at the bottom of the slide at the ground level (Doc. 1).

The part AB of the slide is straight and inclined by an angle  $\alpha = 30^\circ$  with respect to the horizontal. The top A of the slide is situated at a height  $h_A = 1.8$  m above the ground.

Point A is taken as the origin of the x-axis, passing through AB, and of unit vector  $\vec{i}$  (Doc. 2).

The aim of this exercise is to determine the duration of motion of the child from A to B in two cases: without friction and with friction.

Take:

- the horizontal plane passing through B as a reference level for gravitational potential energy;
- $g = 10$  m/s<sup>2</sup>.

1) The child climbs from the ground to point A.

1.1) Calculate the variation of the gravitational potential  $\Delta GPE$  of the system (Child, Earth) between the ground and A.

1.2) Calculate the work  $W$  done by the weight of the child, when he climbs from the ground to A, knowing that  $W = M g (h_i - h_f)$  where  $h_i$  and  $h_f$  are the initial and final heights above the ground.

1.3) Compare  $W$  and  $\Delta GPE$ .

2) Suppose that the child slides without friction from A to B.

2.1) Determine the speed  $V_B$  of the child when he reaches the ground at B.

2.2) Show that the variation of the linear momentum of the child between A and B is  $\Delta \vec{P} = 120 \vec{i}$  (kg.m/s).

2.3) Show that the sum of the external forces exerted on the child, during the downward motion from A to B is  $\Sigma \vec{F}_{ext.} = 100 \vec{i}$  (N).

2.4) Deduce, by applying Newton's second law, the duration  $\Delta t_1$  along AB, knowing that  $\frac{\Delta \vec{P}}{\Delta t} = \frac{d\vec{P}}{dt}$ .

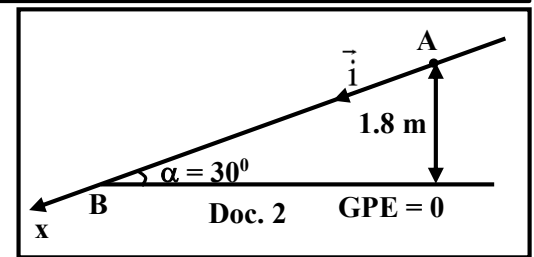
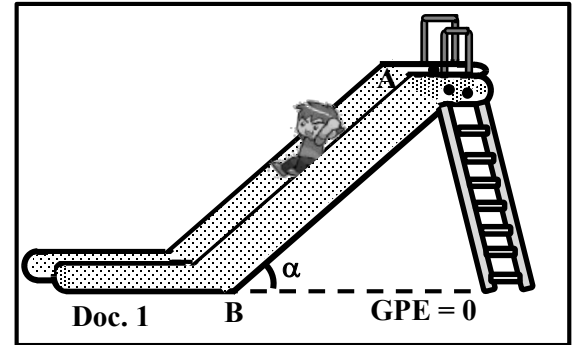
3) In reality, the child is submitted to a force of friction  $\vec{f}$ , supposed constant and parallel to the displacement. During the motion from A to B, the system (Child, Slide, Earth, Atmosphere) loses 25% of its mechanical energy at A.

3.1) Show that during the downward motion of the child from A to B, the variation in the internal energy of the system (Child, Slide, Earth, Atmosphere) is  $\Delta U = 90$  J.

3.2) Deduce that the magnitude of the friction force  $\vec{f}$  is  $f = 25$  N.

3.3) The variation of the linear momentum of the child between A and B, in this case, is  $\Delta \vec{P} = 60\sqrt{3} \vec{i}$  (kg.m/s).

Determine, by applying Newton's second law, the duration  $\Delta t_2$  along AB, knowing that  $\frac{\Delta \vec{P}}{\Delta t} = \frac{d\vec{P}}{dt}$ .



## Exercise 2 (5.5 pts)

### Effect of the capacitance on the duration of discharging of a capacitor

The aim of this exercise is to study the effect of the capacitance of a capacitor on the duration of discharging of a capacitor.

For this aim, we set-up the circuit of document 3 that includes:

- a capacitor, initially uncharged, of adjustable capacitance  $C$ ;
- two identical resistors of resistance  $R = 100 \Omega$ ;
- an ideal battery of voltage  $u_{PN} = E$ ;
- a double switch  $K$ .

#### 1) Charging the capacitor

At  $t_0 = 0$ ,  $K$  is turned to position (1) and the charging process of the capacitor starts. At an instant  $t_1$ , the capacitor is completely charged.

1.1) Indicate the value of the current  $i$  carried by the circuit at  $t_1$ .

1.2) Write, at  $t_1$ , the charge  $Q$  in the capacitor in terms of  $E$  and  $C$ .

#### 2) Discharging the capacitor

The capacitor is completely charged.

At an instant  $t_0 = 0$ , taken as an initial time, the switch  $K$  is turned to position (2); the phenomenon of discharging of the capacitor thus starts.

2.1) Show that the differential equation that describes the variation of the charge  $q$  of plate A of the capacitor

$$\text{is: } R \frac{dq}{dt} + \frac{q}{C} = 0.$$

2.2) The solution of this differential equation is of the

$$\text{form: } q = Q e^{-\frac{t}{\tau}} \text{ where } \tau \text{ is a constant.}$$

Determine the expression of  $\tau$  in terms of  $R$  and  $C$ .

2.3) Calculate the ratio  $\frac{q}{Q}$  at  $t = \tau$ .

2.4) Verify that the capacitor is practically completely discharged at  $t_2 = 5 \tau$ .

#### 3) Duration of discharging a capacitor

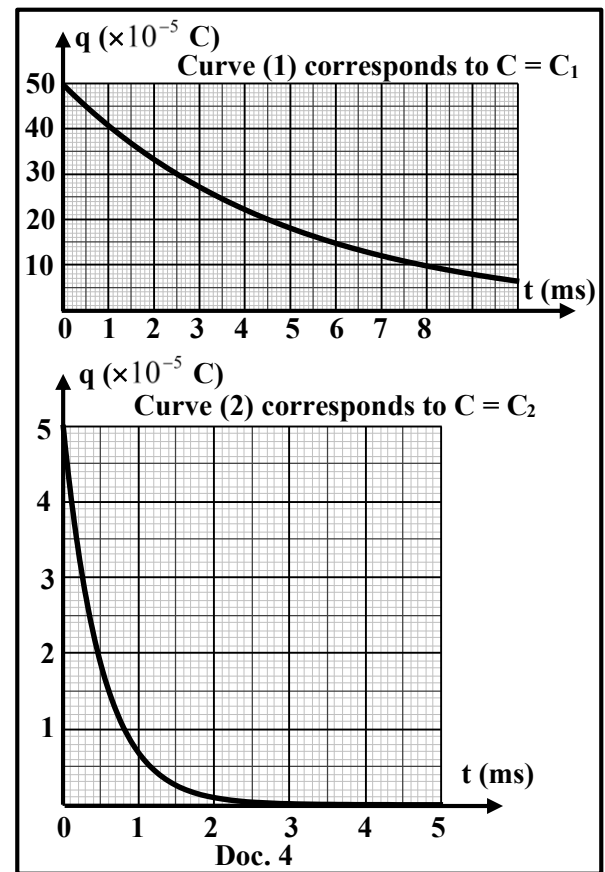
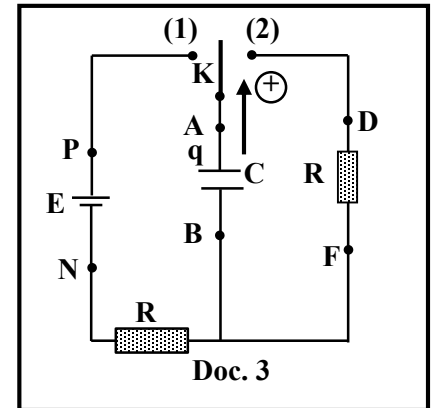
We repeat the charging and the discharging of the capacitor by giving  $C$  two different values  $C_1$  and  $C_2$ . The curves of document 4 show the charge  $q$  during the discharging process of the capacitor for each value of  $C$  as functions of time.

3.1) Using document 4, copy and complete the below table:

	The charge $Q$ (in C) at $t_0 = 0$	The time constant $\tau$ (in ms)
Curve (1) corresponds to $C = C_1$	$Q_1 =$	$\tau_1 =$
Curve (2) corresponds to $C = C_2$	$Q_2 =$	$\tau_2 =$

3.2) Calculate the values  $C_1$  and  $C_2$ .

3.3) Deduce the effect of the capacitance of the capacitor on the duration of the discharging process.



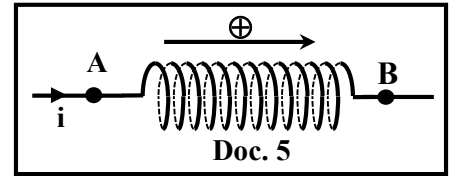
### Exercise 3 (5.5 pts)

#### Inductance and resistance of a coil

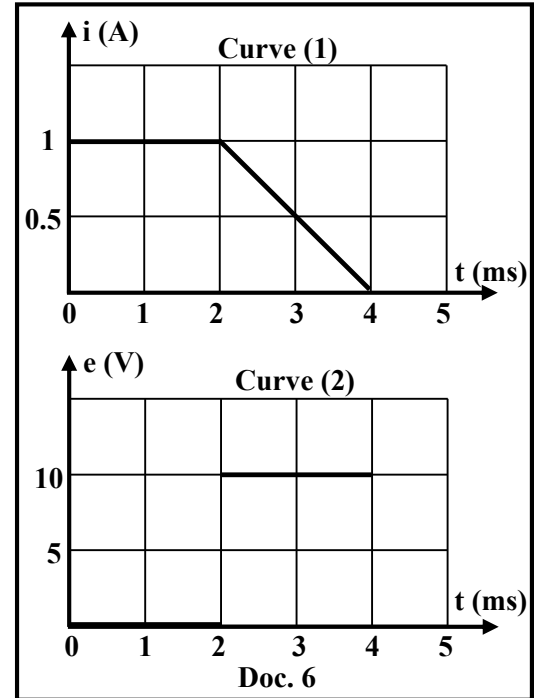
Consider a coil of inductance  $L$  and internal resistance  $r$ .  
The aim of this exercise is to determine  $L$  and  $r$  by two different methods.

#### 1) First method

A portion of a circuit is formed of the coil, that carries a current «  $i$  ». The coil is oriented positively from A to B (Doc. 5).



- 1.1) Write the expression of the self-induced electromotive force «  $e$  » in the coil in terms of  $L$ ,  $i$  and time  $t$ .
- 1.2) The curves (1) and (2) of document 6 show respectively «  $i$  » and «  $e$  » as functions of time, between 0 and 4 ms. Using document 6:



- 1.2.1) Justify each of the following statements:
  - **Statement 1:** between 0 and 2 ms, the coil acts as a resistor of resistance  $r$ .
  - **Statement 2:** between 2 ms and 4 ms, a phenomenon of self-induction takes place in the coil.
  - **Statement 3:** between 2 ms and 4 ms the coil supplies the circuit with the stored magnetic energy.
- 1.2.2) Determine the value of  $L$ .
- 1.2.3) Determine the value of  $r$ , knowing that  $u_{AB} = -5 \text{ V}$  at  $t = 3 \text{ ms}$ .

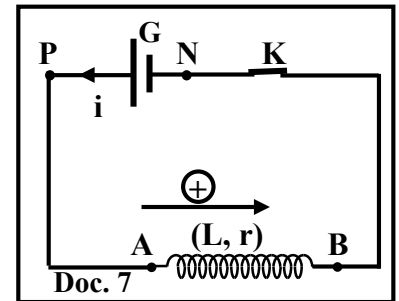
#### 2) Second method

We connect the coil in series with an ideal battery (G) of electromotive force (e.m.f)  $E = 20 \text{ V}$  (Doc.7).

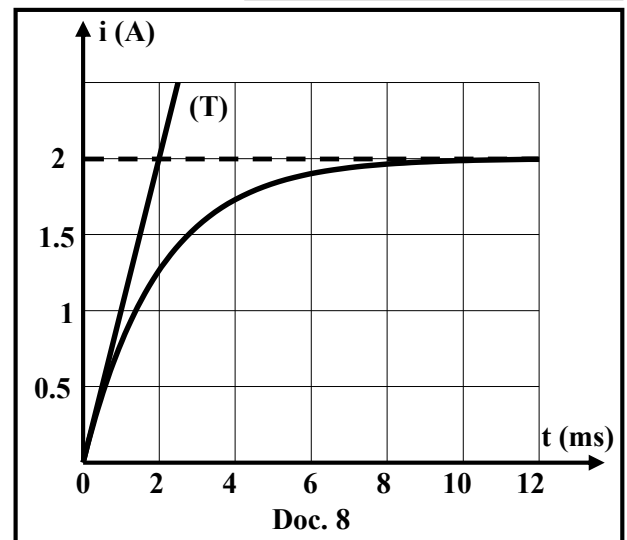
At  $t_0 = 0$ , we close the switch K.

At an instant  $t$ , the circuit carries a current  $i$ .

Document 8 shows the current  $i$  as a function of time and the tangent (T) to the curve  $i(t)$  at  $t_0 = 0$ .



- 2.1) Establish the first order differential equation that describes the variation of the current  $i$  as a function of time.
- 2.2) Determine the expression of the maximum current  $I_m$ , at the steady state, in terms of  $E$  and  $r$ .
- 2.3) Calculate  $r$  using document 8.
- 2.4) Determine, using the differential equation, the expression of  $\frac{di}{dt}$  at  $t_0 = 0$  in terms of  $E$  and  $L$ .
- 2.5) Calculate the slope of the tangent (T). Deduce  $L$ .



## Exercise 4 (4 pts)

### Diameter of a fishing line

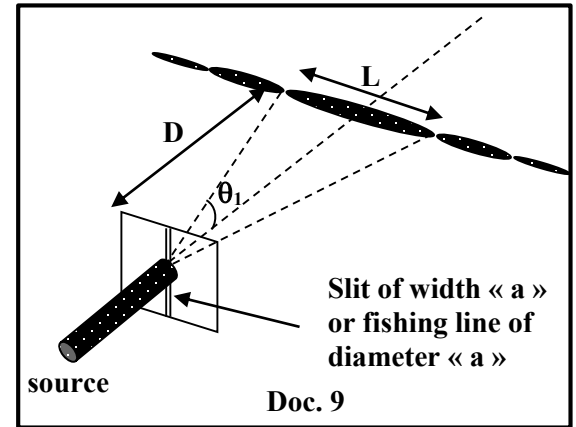
The aim of this exercise is to determine whether the fishing line chosen by a fisherman is suitable to catch the trout fish of a specific size using the phenomenon of diffraction.

#### 1) Set-up of diffraction

A monochromatic light, of wavelength  $\lambda$ , falls normally on a vertical narrow slit of width « a ». The diffraction pattern is observed on a screen placed perpendicularly to the incident light beam at a distance D from the slit.

Let «L» be the linear width of the central bright fringe (Doc. 9).  
The diffraction angles in this exercise are small.

For small angle, take  $\tan \theta \approx \sin \theta \approx \theta$  in radian.



1.1) Describe the diffraction pattern observed on the screen.

1.2) Write, in terms of  $\lambda$  and « a », the expression of the angle of diffraction  $\theta_1$  corresponding to the center of the first dark fringe.

1.3) Show that  $L = \frac{2\lambda D}{a}$ .

#### 2) Diameter of fishing line

A fisherman wants to catch a trout fish of size 50 cm to 55 cm. He bought a thin fishing line made up of 100% copolymer, but the strength of a fishing line also depends on its diameter « a ».

To find out if the chosen fishing line is suitable for such a type of fish, he uses the diffraction set-up of document 9 by replacing the slit of width « a » by the fishing line of diameter « a », so he obtains a diffraction pattern similar to that shown in document 9.

The screen is placed at a distance D from the fishing line, the linear width of the central bright fringe is  $L_1 = 13$  mm. The screen is displaced by 50 cm away from the fishing line, the linear width of the central bright fringe becomes  $L_2 = 19.5$  mm.

2.1) Show that  $D = 1$  m.

2.2) Calculate the diameter « a » of the chosen fishing line, knowing that the wavelength of the used light is  $\lambda = 650$  nm.

2.3) Referring to the table in document 10, specify if the chosen fishing line is suitable for fishing trout fish of size 50 to 55 cm.

Fishing line (100 % copolymer)	Diameter	Use
Fishing line (1)	0.10 mm	It is suitable to fishing a trout fish of size 35 cm to 40 cm.
Fishing line (2)	0.18 mm	It is suitable to fishing a trout fish of size 50 cm to 55 cm.
Fishing line (3)	0.25 mm	It is suitable to fishing a trout fish of size 65 cm to 70 cm.

<https://www.truitesaquaponiques.com/>  
Doc. 10

Exercise 1 (5 pts)		Motion on a slide
Part	Answer	Mark
1.1	The gravitational potential energy of the system at B $GPE_{\text{ground}} = 0$ and at A, $GPE_A = M g h_A = 360 \text{ J}$ $\Delta GPE = GPE_A - GPE_B = 360 - 0 = 360 \text{ J}$	0.75
1.2	The work done by the weight of the child, when he moves from the ground to the top A: $W = M g (h_i - h_f) = 20 \times 10 \times (0 - 1.8) = -360 \text{ J}$	0.25
1.3	$W_{\text{weight}} = -\Delta GPE$	0.25
2.1	$ME_A = GPE_A + KE_A = 360 \text{ J}$ ( $KE_A = 0$ since $V_A = 0$ ) In the absence of friction, (or the work done by the nonconservative forces is zero) the mechanical energy of the system is conserved, then $ME_B = ME_A = 360 \text{ J}$ But $ME_B = KE_B + GPE_B$ ; $GPE_B = 0$ (on reference) So $\frac{1}{2} M V_B^2 = 360$ therefore $V_B = 6 \text{ m/s}$	0.75
2.2	$\Delta \vec{P} = \vec{P}_B - \vec{P}_A$ ; $\Delta \vec{P} = M \vec{V}_B - M \vec{V}_A$ , therefore $\Delta \vec{P} = 20 \times 6 \vec{i} - \vec{0}$ , then $\Delta \vec{P} = 120 \vec{i}$	0.5
2.3	$\Sigma \vec{F}_{\text{ext}} = M \vec{g} + \vec{N}$ , along $\vec{i}$ : $\Sigma \vec{F}_{\text{ext}} = Mg \cdot \sin \alpha \vec{i} + \vec{0} = 100 \vec{i}$	0.5
2.4	$\Delta \vec{P} = \Sigma \vec{F}_{\text{Ext}} \times \Delta t_1$ , so $120 \vec{i} = 100 \vec{i} \times \Delta t_1$ , then $\Delta t_1 = 1.2 \text{ s}$	0.25
3.1	The system (Child, Slide, Earth, Atmosphere) is energetically isolated, So its total energy $E = ME + U = \text{constant}$ So, $\Delta U = -\Delta(ME)$ There is a loss of 25 % of ME; so $\Delta(ME) = -0.25 \times ME_A = -0.25(360) = -90 \text{ J}$ : Hence, $\Delta U = 90 \text{ J}$	0.75
3.2	The variation in mechanical energy equals the work of friction: $\Delta ME = W_{\vec{f}}$ so $\Delta(ME) = -90 = -f \times AB = -f \times \frac{h_A}{\sin(\alpha)}$ $-90 = -f \times 3.6$ , thus $f = 25 \text{ N}$	0.5
3.3	$\Delta \vec{P} = \Sigma \vec{F}_{\text{ext}} \times \Delta t_2$ , $\Sigma \vec{F}_{\text{Ext}} = (Mg \cdot \sin \alpha - f) \vec{i} + \vec{0}$ so $60 \sqrt{3} \vec{i} = (100 - 25) \vec{i} \times \Delta t_2$ , then $\Delta t_2 = 1.385 \text{ s}$	0.5

Exercise 2 (5.5 pts) Effect of the capacitance on the discharging of a capacitor											
Part	Answer	Mark									
1.1	At instant $t_1$ : $i = 0$	0.25									
1.2	$Q = C E$	0.25									
2.1	$u_C = u_{DF} = u_R$ ; $\frac{q}{C} = R i$ ; But $i = -\frac{dq}{dt}$ so we get : $R \frac{dq}{dt} + \frac{q}{C} = 0$	0.5									
2.2	$q = Q e^{-\frac{t}{\tau}}$ ; $\frac{dq}{dt} = -\frac{Q}{\tau} e^{-\frac{t}{\tau}}$ we substitute $q$ and $\frac{dq}{dt}$ in the differential equation $-R \frac{Q}{\tau} e^{-\frac{t}{\tau}} + \frac{Q e^{-\frac{t}{\tau}}}{C} = 0$ ; $Q e^{-\frac{t}{\tau}} \left[-\frac{R}{\tau} + \frac{1}{C}\right] = 0$ ; But $Q e^{-\frac{t}{\tau}} \neq 0$ ; so $-\frac{R}{\tau} + \frac{1}{C} = 0$ ; therefore $\frac{R}{\tau} = \frac{1}{C}$ thus $\tau = RC$	1									
2.3	the ratio: $\frac{q}{Q} = \frac{Q e^{-\frac{t}{\tau}}}{Q}$ at $t = \tau$ : $\frac{q}{Q} = e^{-1} = 0.37$	0.5									
2.4	$q = Q e^{-\frac{t}{\tau}}$ ; à $t_2 = 5 RC$ : $q = Q e^{-5} = 0.006 Q \approx 0$	0.5									
3.1	Curve (1): At $t_0 = 0$ : $Q_1 = 50 \times 10^{-5} \text{ C}$ at $t = \tau_1$ : $q = 0.37 \times 50 \times 10^{-5} = 18.5 \times 10^{-5} \text{ C}$ . From the graph: $q = 18.5 \times 10^{-5} \text{ C}$ at $t = 5 \text{ ms}$ ; so $\tau_1 = 5 \text{ ms}$  Curve (2): at $t_0 = 0$ : $Q_2 = 5 \times 10^{-5} \text{ C}$ at $t = \tau_2$ : $q = 0.37 \times 5 \times 10^{-5} = 1.85 \times 10^{-5} \text{ C}$ . From the graph: $q = 1.85 \times 10^{-5} \text{ C}$ at $t = 0.5 \text{ ms}$ ; so $\tau_2 = 0.5 \text{ ms}$	1									
	<table border="1"> <thead> <tr> <th></th> <th>Charge <math>Q</math> at <math>t_0 = 0</math></th> <th>The time constant <math>\tau</math></th> </tr> </thead> <tbody> <tr> <td>Curve (1) for <math>C = C_1</math></td> <td><math>Q_1 = 50 \times 10^{-5} \text{ C}</math></td> <td><math>\tau_1 = 5 \text{ ms}</math></td> </tr> <tr> <td>Curve (2) for <math>C = C_2</math></td> <td><math>Q_2 = 5 \times 10^{-5} \text{ C}</math></td> <td><math>\tau_2 = 0.5 \text{ ms}</math></td> </tr> </tbody> </table>		Charge $Q$ at $t_0 = 0$	The time constant $\tau$	Curve (1) for $C = C_1$	$Q_1 = 50 \times 10^{-5} \text{ C}$	$\tau_1 = 5 \text{ ms}$	Curve (2) for $C = C_2$	$Q_2 = 5 \times 10^{-5} \text{ C}$	$\tau_2 = 0.5 \text{ ms}$	
	Charge $Q$ at $t_0 = 0$	The time constant $\tau$									
Curve (1) for $C = C_1$	$Q_1 = 50 \times 10^{-5} \text{ C}$	$\tau_1 = 5 \text{ ms}$									
Curve (2) for $C = C_2$	$Q_2 = 5 \times 10^{-5} \text{ C}$	$\tau_2 = 0.5 \text{ ms}$									
3.2	$\tau_1 = R C_1$ , so $C_1 = \frac{\tau_1}{R}$ : $C_1 = \frac{5 \times 10^{-3}}{100}$ thus $C_1 = 5 \times 10^{-5} \text{ F} = 50 \mu\text{F}$ $\tau_2 = R C_2$ , so $C_2 = \frac{\tau_2}{R}$ ; $C_2 = \frac{0.5 \times 10^{-3}}{100}$ thus $C_2 = 0.5 \times 10^{-5} \text{ F} = 5 \mu\text{F}$	1									
3.3	As the capacitance increases , the duration of discharging increases	0.5									

Exercise 3 (5.5 pts)		Characteristics of a coil
Part	Answer	Mark
1.1	$e = -L \frac{di}{dt}$	0.25
1.2.1	<ul style="list-style-type: none"> <li>Statement 1 : During this interval <math>i = \text{constant}</math>, so <math>\frac{di}{dt} = 0</math> thus <math>e = 0</math>, The voltage across the coil <math>u_{AB} = ri - e = ri</math> The coil acts as a resistor of resistance <math>r</math>.</li> <li>Statement 2 : <math>i</math> varies with time so <math>e \neq 0</math> therefore <math>e</math> exists this implies that a phenomenon of self-induction appears in the circuit.</li> <li>Statement 3 : <math>i</math> decreases, then <math>W_{\text{mag}} = \frac{1}{2} Li^2</math> decreases Or <math>e \cdot i &gt; 0</math>, then it acts as a generator</li> </ul>	1.5
1.2.2	<p>Between 2 ms and 4 ms : <math>e = 10 \text{ V}</math></p> $\frac{di}{dt} = \text{slope} = \frac{0-1}{4 \times 10^{-3} - 2 \times 10^{-3}} = -500 \text{ A/s}$ <p><math>e = -L \frac{di}{dt}</math> so : <math>10 = -L(-500)</math>, thus <math>L = 0.02 \text{ H} = 20 \text{ mH}</math></p>	0.75
1.2.3	<p><math>u_{AB} = ri - e</math>; at <math>t = 3 \text{ ms}</math> : <math>-5 = r(0.5) - 10</math> <math>0.5r = 10 - 5 = 5</math>, then <math>r = 10 \Omega</math></p>	0.5
2.1	$u_g = u_L$ ; $E = ri + L \frac{di}{dt}$ ;	0.5
2.2	<p>in steady state : <math>i = I_m</math>; and <math>\frac{di}{dt} = 0</math></p> <p>so <math>E = r I_m</math>, thus <math>I_m = \frac{E}{r}</math></p>	0.5
2.3	$I_m = 2 \text{ A}$ ; $2 = \frac{20}{r}$ , then $r = 10 \Omega$	0.25
2.4	<p><math>E = ri + L \frac{di}{dt}</math>, then <math>\frac{di}{dt} = \frac{E - ri}{L}</math> ;</p> <p>At <math>t_0 = 0</math>: <math>i = 0</math> then <math>\left. \frac{di}{dt} \right _{t_0=0} = \frac{E}{L}</math></p>	0.5
2.5	<p>Slope of the tangent = <math>\frac{2}{2 \times 10^{-3}} = 1000 \text{ A/s}</math></p> <p>But, slope of the tangent = <math>\left. \frac{di}{dt} \right _{t_0=0} = \frac{E}{L}</math> ;</p> <p>So, <math>1000 = \frac{20}{L}</math>, then <math>L = 0.02 \text{ H} = 20 \text{ mH}</math></p>	0.75

Exercise 4 (4 pts)		Diameter of a fishing line	
Part	Answer	Mark	
1.1	<p>We observe on the screen:</p> <ul style="list-style-type: none"> <li>✓ Alternating bright and dark fringes;</li> <li>✓ The central bright fringe is the most intense and has a width double that of the other bright fringes;</li> <li>✓ The direction of the fringes is perpendicular to the direction of the slit.</li> </ul>	0.75	
1.2	$\sin\theta_1 \approx \theta_1 = \frac{\lambda}{a}$	0.25	
1.3	$\tan \theta_1 = \frac{L/2}{D}$ , then $\theta_1 = \frac{L}{2D}$ ; $\frac{\lambda}{a} = \frac{L}{2D}$ ; thus, $L = \frac{2\lambda D}{a}$	1	
2.1	$\frac{\lambda}{a} = \frac{L_1}{2D_1} = \frac{L_2}{2D_2}; \quad \frac{L_2}{L_1} = \frac{D_2}{D_1} = \frac{D+0.5}{D}$ $\frac{L_2}{L_1} = \frac{D+0.5}{D}, \quad \text{so} \quad \frac{19.5}{13} = \frac{D+0.5}{D}$ <p>then <math>19.5 D = 13 D + 6.5</math>; thus <math>D = 1 \text{ m}</math></p>	1	
2.2	$a = \frac{2\lambda D}{L_1}$ then $a = \frac{2 \times 650 \times 10^{-9} \times 1}{1.3 \times 10^{-2}}$ ; $a = 0.1 \text{ mm}$	0.5	
2.3	The chosen line is not suitable to catch the trout fish of size 50 to 55 cm. Because the diameter of the line is less than 0.18 mm.	0.5	