مسابقة في مادة الفيزياء | الاسم:
المدة: ثلاث ساعات | الرقم:

This exam is formed of four obligatory exercises in four pages.
The use of non-programmable calculators is recommended.

Exercise 1 (7.5 points) Two periodic motions of a system

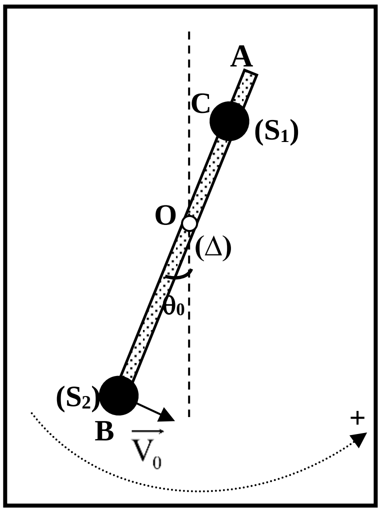
Consider a system (S) formed by a uniform rigid thin rod (AB) of length $\ell = 0.6$ m and mass M = 1 kg, and two identical particles (S_1) and (S_2) each of mass m = 0.5 kg. (S_2) is fixed at one end B of the rod whereas (S_1) is fixed, on the rod, at an adjustable distance from its midpoint O.

Let G be the center of (S) with $\overline{OG} = a$.

The system is shifted in a vertical plane by an angle $\theta_0 = -0.08$ rad from its stable equilibrium position ($\theta = 0$), and then it is launched with an initial angular velocity $\theta'_0 = 0.3$ rad/s at $t_0 = 0$. The system thus moves without friction in the vertical plane about a horizontal axis (Δ) passing through O.

At an instant t, the position of the rod is denoted by its angular abscissa θ and its angular velocity $\theta' = \frac{d\theta}{dt}$.

The moment of inertia of the rod about (Δ) is $I_R = \frac{1}{12}M\ell^2$.



Doc. 1

The aim of this exercise is to study two periodic motions of this system corresponding to two different positions of (S_1).

1) First motion

(S_1) is fixed at a point C such that $\overline{OC} = -0.2$ m (Doc. 1). The formed system is a compound pendulum that oscillates with a maximum angle $\theta_m < 10^o$.

Given: $g = 10$ m/s^2 ; $\cos\theta \cong 1 - \frac{\theta^2}{2}$ and $\sin\theta \cong \theta$ (θ in rad) for $\theta \le 10^o$.

1-1) Prove that a = 0.025 m.

1-2) Calculate the moment of inertia I of the pendulum about (Δ).

1-3) Name the external forces acting on (S).

1-4) Given that the moment of the weight of the system about (Δ) is: $\mathcal{M} = -(2m + M)\, g\, a \sin\theta$.
Apply the theorem of angular momentum, to prove that the differential equation that governs the variation of θ is $\theta'' + \frac{(2m+M)ga}{I}\theta = 0$.

1-5) Deduce that the motion of the pendulum is periodic and show that
its period T_0 does not depend on θ'_0.

1-6) Calculate T_0.

1-7) The expression of the angular abscissa of the pendulum is
$\theta = \theta_m \sin(\frac{2\pi}{T_0}t + \varphi)$. Deduce the values of θ_m and φ.

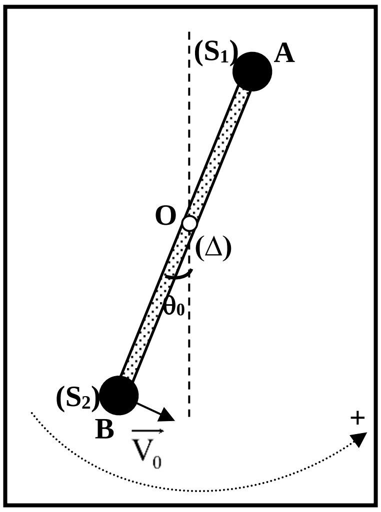
2) Second motion

(S_1) is fixed at the point A such that $\overline{OA} = -0.3$ m (Doc. 2).

2-1) Prove that G coincides with point O (a = 0).

2-2) Using the differential equation established in part (1-4), show that the system (S) is now in uniform rotational motion.

2-3) The period T of the motion of the system (S) depends on θ'_0.
Write the relation between T and θ'_0. Calculate the value of T.



Doc. 2

Exercise 2 (7.5 points) RLC series circuit

A capacitor of capacitance $C = 6\ \mu F$, a purely inductive coil of inductance $L = 0.8$ H, and a resistor of resistance $R = 100\ \Omega$, are connected in series between two points A and F.

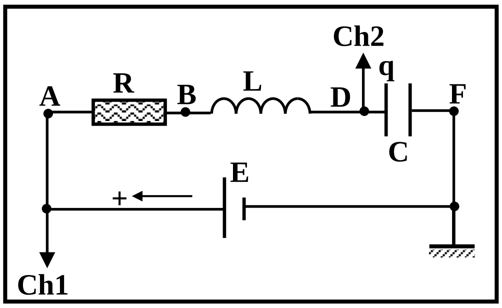
The aim of this exercise is to determine the electric energy consumed by this circuit when fed by two different voltages.

1) RLC circuit fed by a constant voltage

The circuit is connected across an ideal battery of e.m.f $E = u_G = u_{AF} = 10$ V (Doc. 3).

The capacitor is initially uncharged. An oscilloscope is connected in the circuit in order to display the voltages u_{AF} and $u_{DF} = u_C$ across the battery and the capacitor respectively.

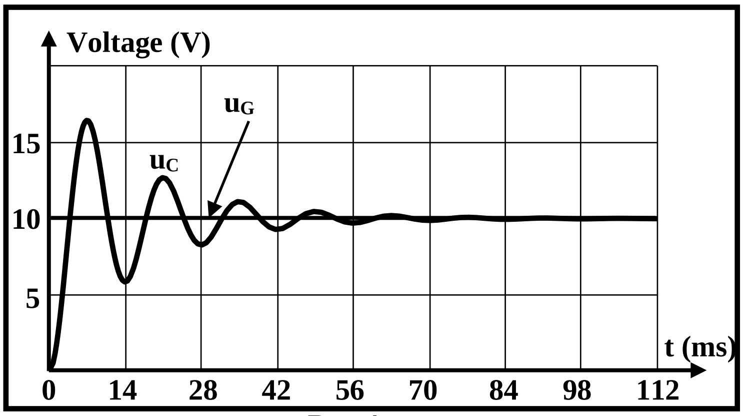
Document (4) is a graph that shows the voltages u_G and u_C as functions of time.



Doc. 3

1-1) Apply the law of addition of voltages to show that the second order differential equation of u_C:

$$u''_C + \frac{R}{L}u'_C + \frac{u_C}{LC} = \frac{E}{LC}.$$



Doc.4

1-2) Use document (4) to answer the following questions:

1-2-1) Choose, among a), b) and c) the correct expression.

- **a)** u_C is alternating sinusoidal;
- **b)** u_C oscillates and finally becomes zero;
- **c)** u_C oscillates about E and then becomes equal to E.

1-2-2) Determine the angular frequency of oscillations of u_C.

1-2-3) Specify a time interval during which the capacitor consumes energy and a time interval during which the capacitor supplies energy.

1-3) After a certain time, the steady state is attained.

1-3-1) Prove, using document (4), that the capacitor neither consumes nor supplies energy;

1-3-2) Calculate the electric energy stored in the capacitor;

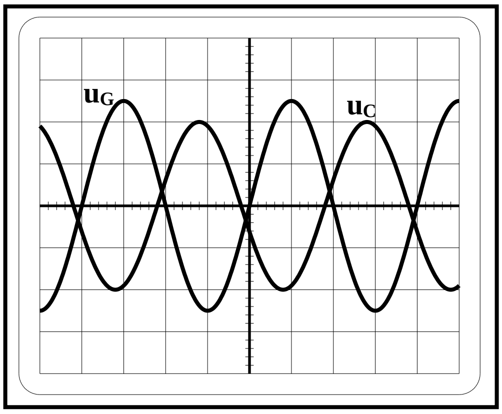
1-3-3) Determine the value of the current in the circuit;

1-3-4) Deduce that the circuit neither consumes nor supplies electric energy.

2) RLC circuit fed by an alternating sinusoidal voltage

The battery is replaced by a function generator (G) providing an alternating sinusoidal voltage. The voltage across the terminals of (G) is $u_G = u_{AF} = 10 \sin(215\pi t)$ (SI) and the voltage across the capacitor in the steady state is $u_C = U_{C(m)} \sin(215\pi t + \varphi)$ (SI)

The curves of document (5) represent, in the steady state, the voltages u_G and u_C displayed on the screen of the oscilloscope. The vertical sensitivity on both channels is $S_V = 4$ V/div.



Doc. 5

2-1) Indicate the value of the angular frequency of u_C.

2-2) u_C performs forced electromagnetic oscillations. Justify.

2-3) Calculate, using document 5, $U_{C(m)}$ and $|\varphi|$.

2-4) Deduce that the expression of the current in the circuit is $i = 0.032 \cos(215\pi t - 2.83)$ (SI).

2-5) Determine the average power consumed by the circuit.

2-6) Deduce the electrical energy consumed by the circuit during one period of the voltage.

Exercise 3 (7.5 points) **Identification of two monochromatic lights**

Consider a dichromatic light source (S) emitting two monochromatic lights (A) and (B) of wavelengths in air λ_1 and λ_2 respectively. The color of the light beam emitted from (S) appears magenta.

The aim of this exercise is to determine λ_1 and λ_2.

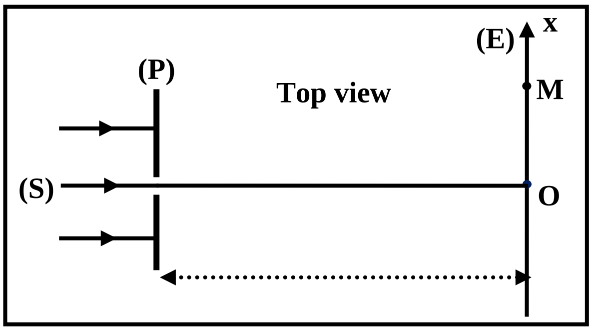
Given:

- The range of wavelength of visible light in air: $400 \text{ nm} \leq \lambda \leq 800 \text{ nm}$;
- $\lambda_1 < \lambda_2$.

1) Diffraction of light

(S) illuminates in air, at normal incidence, a vertical thin slit, of width a = 0.2 mm, which is cut in an opaque plane (P). We observe a diffraction pattern on a screen (E) placed parallel to (P) at a distance D = 2 m away from it. A point M on the screen belongs to the obtained diffraction pattern, and it has a position $x = \overline{OM}$ relative to the center O of the central bright fringe (Doc. 6).

The diffraction angles of the fringes in the following questions are small.



Doc. 6

1-1) A filter is placed in front of source (S). It transmits light (B) only of wavelength $\lambda = \lambda_2$.
Let M be the center of a dark fringe of order n (n is integer)

1-1-1) Write in terms of a, n, and λ, the expression of the diffraction angle θ of M.

1-1-2) Prove that the abscissa of M O is $x = \frac{n\lambda D}{a}$.

1-1-3) The central bright fringe obtained by the diffraction of a monochromatic light of wavelength λ separates the centers of two dark fringes.

Deduce using part (1-1-2) that the width of the central bright fringe is $L = \frac{2\lambda D}{a}$.

1-2) We remove the filter, so that the two lights (A) and (B) reach the screen.

1-2-1) Compare the width of the central bright fringe obtained by the diffraction of light (A) to that obtained by the diffraction of light (B).

1-2-2) We notice that the central fringe on the diffraction pattern appears magenta. The width of this fringe is 9.3 mm. Deduce that $\lambda_1 = 465$ nm.

1-3) The two lights (A) and (B) still reaching the screen. The abscissa of a point Q in the diffraction pattern is x = 27.9 mm. Q is the center of a dark fringe of order n_1 for light (A) and at the same time Q is the center of a dark fringe of order n_2 for light (B).

1-3-1) Determine the value of n_1.

1-3-2) Prove that $n_2 \times \lambda_2 = 2790$ nm.

1-3-3) Prove that $4 \leq n_2 < 6$.

1-3-4) Deduce that the possible values of λ_2 are 558 nm and 697.5 nm.

2) Photoelectric effect

The filter is placed again in front of the source (S). The Light (B) of wavelength λ_2 emitted from this filter illuminates a pure cesium plate of threshold wavelength $\lambda_0 = 590$ nm.

A convenient apparatus is placed near the plate in order to detect the electrons ejected from the cesium plate. No emission of photoelectrons from cesium takes place. Deduce λ_2.

Exercise 4 (7.5 points) Radioactive human body

Our bodies contain trace amounts of some radionuclides because we eat, drink, and breathe radioactive substances that are naturally present in the environment.
Document 7 represents the mass and the activity of natural radionuclides found in a human body of mass m = 70 kg based on a research center publication.
The aim of this exercise is to specify whether these radionuclides have harmful effects on the human body.

Nuclide	Mass	Activity
Potasium 40	17 mg	4.4 kBq
Uranium	90 µg	1.1 Bq
Thorium	30 µg	0.11 Bq
Carbon 14	22 ng	3.63 kBq
Radium	31 pg	1.1 Bq
Polonium	0.2 pg	37 Bq
Tritium	0.06 pg	23 Bq

Doc. 7

Given: $m({}^{14}_{6}C) = 13.99995$ u ; $m({}^{A}_{Z}N) = 13.99923$ u ;

$m_{(e^-)} = 5.486\times10^{-4}$ u; $1\text{ u} = 931.5\,\frac{\text{MeV}}{c^2}$, $1\text{ u} = 1.66\times10^{-27}$ kg ; $1\text{ eV} = 1.6\times10^{-19}$ J.

1) Decay of carbon 14

Carbon-14 is a beta-minus emitter. Its decay equation is:

$${}^{14}_{6}C \rightarrow {}^{A}_{Z}N + {}^{M}_{D}e + X$$

1-1) Particle X is an antineutrino. Why?

1-2) Complete with justification the above equation.

1-3) Define the activity of a radioactive sample.

1-4) Carbon 14 is constantly renewed by ingestion. Deduce that the activity of carbon 14 inside the person's body remains constant with time.

1-5) Use document 7 to:

1-5-1) calculate the number N_0 of ${}^{14}_{6}C$ nuclei present in the person's body. (1ng $=10^{-9}$ g)

1-5-2) determine the half-life (in years) of carbon 14.

1-6) Determine, in MeV and in J, the energy liberated by the decay of one nucleus of carbon 14. Neglect the mass of particle X.

1-7) Prove that the energy liberated during one year by the decay of carbon 14 inside the person's body is $E' \cong 2.94\times10^{-3}$ J.

2) Effects of the radiation on the human body

2-1) The person's body does not absorb the energy of the antineutrino.
Give a property of the antineutrino that justifies this statement.

2-2) Knowing that:

- the physiological equivalent of dose, in Sv, received by the person's body of mass m from the decay of carbon 14 in one year is $ED_1 = \frac{E'}{m} \times QF$, where QF = 1 is the quality factor of the beta-minus radiation;
- the physiological equivalent of dose received by the person's body in one year from the decay of the other radionuclides found in the body is $ED_2 = 0.268$ mSv;
- the total physiological equivalent of dose received by the person's body is $ED = ED_1 + ED_2$;
- the maximum permissible physiological equivalent of dose received by the human body is 5 mSv per year.

2-2-1) Calculate the total physiological equivalent of dose received by this person's body in one year.

2-2-2) Deduce if these radionuclides are dangerous for this person in one year.

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المديريّة العامّة للتربية | فرع العلوم العامّة | السبت 26 أيلول 2020

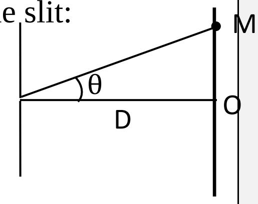
دائرة الامتحانات الرسميّة

مسابقة في مادة الفيزياء | الاسم:

المدة: ثلاث ساعات | الرقم:

First Exercise		Solution	7.5 pts
	1	$a = \frac{m\,x_A + m\,x_B + M(0)}{m+m+M} = \frac{0.5\times(-0.2)+(0.5)(0.3)+0}{0.5+0.5+1}$ **= 0.025 m**	0.5 0.25
	2	$I = I_R + I_A + I_B = \frac{1}{12}M\ell^2 + md^2 + m(OB)^2 = \frac{1}{12}\times 1\times 0.6^2 + 0.5(0.3^2+0.2^2)$ I = **0.095 kgm²**	0.75
	3	Weight $\vec{W}$ of the system, and the support reaction $\vec{R}$ at O.	0.25
	4	$\Sigma\mathcal{M}_{ext} = \frac{d\sigma}{dt} = \frac{d(I\theta')}{dt} = I\,\theta''$, so $\mathcal{M}_{\vec{W}} + \mathcal{M}_{\vec{R}} = I\,\theta''$, but $\mathcal{M}_{\vec{R}} = 0$ since $\vec{R}$ intersects (Δ) $-(2m+M)g\,a\sin\theta = I\,\theta''$, so $-(2m+M)g\,a\,\theta = I\,\theta''$, hence $\boldsymbol{\theta'' + \frac{(2m+M)g\,a}{I}\theta = 0}$	0.75
	5	The differential equation is of the form $\theta'' + \omega_o^2\,\theta = 0$ **,** where $\omega_o^2 = \frac{(2m+M)g\,a}{I}$ is a positive constant **,** then, the motion is simple harmonic so it is periodic. $T_o = \frac{2\pi}{\omega_o} = \boldsymbol{2\pi\sqrt{\frac{I}{(2m+M)ga}}}$, so it **does not depend** on the value of θ'_o.	0.5 0.75
	6	$T_o = 2\pi\sqrt{\frac{0.095}{(2\times0.5+1)\times10\times0.025}}$ = **2.74 s**	0.25
	7	θ_o = - 0.08 = $\theta_m\sin(\varphi)$, then $\sin(\varphi) = \frac{-0.08}{\theta_m}$ eq (1) $\theta' = \frac{2\pi\,\theta_m}{T_o}\cos(\frac{2\pi}{T_o}t+\varphi)$, so $0.3 = \frac{2\pi\,\theta_m}{2.74}\cos(\varphi)$, then $\cos(\varphi) = \frac{0.131}{\theta_m}$ eq (2) $\sin^2\varphi + \cos^2\varphi = 1$, then $\left(\frac{0.08}{\theta_m^2}\right)^2 + \left(\frac{0.131}{\theta_m^2}\right)^2 = 1$; therefore, $\boldsymbol{\theta_m}$ **= 0.153 rad** $\sin(\varphi) = \frac{-0.08}{0.153}$ = -0.5228 , so φ = -0.55 rad or φ = 3.69 rad From eq (2) cos φ > 0; therefore , **φ = -0.55 rad**	0.75 1
	1	$a = \frac{0.5(0.3)+0.5(-0.3)+0}{2}$ = **0**	0.25
	2	$\theta'' + \frac{(2m+M)g\,a}{I}\theta = 0$, for a = 0 , then $\theta'' = 0$ Since $\theta'_o \neq 0$, therefore the motion is uniform rotational .	0.75
	3	$T = \frac{2\pi}{\theta'_o}$, $T = \frac{2\pi}{0.3}$ = **20.9 s**	0.5 0.25

Second Exercise			Solution	7.5 pts
1	1		$u_{AF} = u_{AB} + u_{BD} + u_{DF}$, then $E = Ri + L\frac{di}{dt} + u_C$, but $i = \frac{dq}{dt} = C\frac{du_C}{dt}$	0.75
	2	1	Answer c) u_C oscillates about E and then becomes equal to E.	0.25
		2	T = 14 ms , then $\omega = \frac{2\pi}{T} = \frac{2\pi}{14 \times 10^{-3}}$ = **448.8 rad/s**	0.75
		3	$W_C = \frac{1}{2}Cu_C^2$, so the capacitor consumes energy when $\lvert u_C \rvert$ increases During **] 0, 7 ms[**: $\lvert u_C \rvert$ increases , then the capacitor consumes energy The capacitor consumes energy when $\lvert u_C \rvert$ decreases During **]7ms, 14 ms[**: $\lvert u_C \rvert$ decreases , then the capacitor supplies energy	0.25 0.25 0.25
	3	1	The voltage across the capacitor becomes equal to E which is constant	0.25
		2	$W_C = \frac{1}{2}Cu_C^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 10^2 = \mathbf{3 \times 10^{-4}\ J}$	0.5
		3	$i = C\frac{du_C}{dt}$, since u_C = E = constant , then **i = 0**	0.5
		4	$W_L = \frac{1}{2}Li^2$ and the thermal power of R is $P_R = R\,i^2$ In the steady state, i = 0 ; therefore, $W_R = W_L = 0$ $W_{C(Consumed)} = 0$, then the circuit neither consumes nor supplies electric energy.	0.5
2	1		$\omega = 215\ \pi$ rad/s	0.25
	2		The angular frequency of u_C is equal to that of the function generator	0.25
	3		$U_{C(m)} = S_V \times Y_m = 4 \times 2 =$ **8 V** $\lvert\varphi\rvert = \frac{2\pi d}{D} = \frac{2\pi \times 1.8}{4}$ = **2.83 rad**	0.25 0.25
	4		u_C lags behind u_G by $\lvert\varphi\rvert$, then $u_C = U_{C(m)}\sin(215\pi t - \lvert\varphi\rvert) = 8\sin(215\pi t - 2.83)$ $i = C\frac{du_C}{dt} = 6 \times 10^{-6} \times 215\ \pi \times 8\cos(215\pi t - 2.83) = 0.032$	0.75
	5		$i = 0.032\cos(215\pi t - 2.83) = 0.032\sin(215\pi t - 2.83 + \frac{\pi}{2})$ $i = 0.032\sin(215\pi t - 1.26)$ $P_{av} = I_{eff}\,U_{G(eff)}\cos\beta = \frac{0.032}{\sqrt{2}} \times \frac{10}{\sqrt{2}}\cos 1.26$ = **0.049 W**	0.75
	6		$T = \frac{2\pi}{\omega} = \frac{2\pi}{215\ \pi} = 9.3 \times 10^{-3}$ s $W = P_{av} \times T = 0.049 \times 9.3 \times 10^{-3} = \mathbf{4.6 \times 10^{-4}\ J}$	0.75

Third Exercise	Solution	7.5 pts
	$\sin\theta = \frac{n\lambda}{a}$, since the angles are small then $\sin\theta \cong \theta$, then $\boldsymbol{\theta = \frac{n\lambda}{a}}$	**0.5**
	Consider the right triangle formed by O, M, and the center of the slit: $\tan\theta \cong \theta = \frac{x}{D}$, then $x = \theta\, D = \boldsymbol{\frac{n\,\lambda\, D}{a}}$ 	**1**
	The first dark fringes situated on both sides of the central bright fringe, then $L = x_1 - x_{-1} = \frac{\lambda D}{a} - \frac{-\lambda D}{a} = \boldsymbol{\frac{2\lambda D}{a}}$	**1**
	Since $\lambda_1 < \lambda_2$, then $L_1 < L_2$ Therefore, the width of the central bright fringe of (B) is longer than that of (A). The width of the central bright fringe of (B) is longer than that of (A) The color appears magenta in the common area of the central fringes of (A) and	**0.25** **0.5**
	$L_1 = \frac{2\,\lambda_1\, D}{a}$, then $\lambda_1 = \frac{a\, L_1}{2D} = \frac{0.2\times10^{-3}\times 9.3\times10^{-3}}{2\times2} = 4.65\times10^{-7}$ m = **465 nm**	**0.5**
	$x = \frac{n_1\,\lambda_1 D}{a}$, then $n_1 = \frac{a\,x}{D\,\lambda_1} = \frac{0.2\times10^{-3}\times27.9\times10^{-3}}{2\times465\times10^{-9}} = \mathbf{6}$	**0.5**
	$x = \frac{n_2\,\lambda_2 D}{a}$, then $n_2\,\lambda_2 = \frac{a\,x}{D} = \frac{0.2\times10^{-3}\times27.9\times10^{-3}}{2}$, hence $\boldsymbol{n_2\,\lambda_2 = 2.79\times10^{-6}}$ **m** Or: $\boldsymbol{n_2\,\lambda_2}$ **= 2790 nm**	**0.75**
	$n_2 = \frac{2790}{\lambda_2}$ and $465 \text{ nm} < \lambda_2 \le 800$ nm $n_2 \ge \frac{2790}{800}$, so $n_2 \ge 3.49$ but $n \in N$; therefore, $n_2 \ge 4$ $n_2 < \frac{2790}{400}$; therefore $\boldsymbol{n_2 < 6}$ or $n_2\,\lambda_2 = n_1\,\lambda_1$, but $\lambda_2 > \lambda_1$, then $n_2 < n_1$; therefore $\boldsymbol{n_2 < 6}$	**1**
	$\lambda_2 = \frac{a.x}{n_2 D}$ if $n_2 = 4$ then $\lambda_2 =$ **697,5 nm** if $n_2 = 5$ then $\lambda_2 =$ **558 nm**	**0.75**
2	Photoelectrons takes place if $\lambda_2 \le \lambda_o$, but we have no emission of electrons, then $\lambda_2 > \lambda_o$; therefore , $\boldsymbol{\lambda_2}$ **= 697.5 nm**	**0.75**

Fourth Exercise			Solution	7.5 pts
1	1		Since the type of radioactivity is β^- Or : Since in radioactivity the emission of the electron is accompanied by the emission of an antineutrino.	**0.25**
	2		Law of conservation of mass number: 14 = A + 0 + 0 , then A = 14 Law of conservation of charge number: 6 = Z - 1 + 0 , then Z = 7	**0.5** **0.5** **0.25**
	3		Activity is the number of disintegrations per unit time.	**0.5**
	4		A = λ N. Since λ and N are constant with time , then A remains constant with time	**0.5**
	5	1	$N_o = \frac{m}{m\ {}^{14}_{6}C} = \frac{22 \times 10^{-9}}{13.99995 \times 1.66 \times 10^{-24}} = \mathbf{9.466 \times 10^{14}}$ **nuclei**	**0.75**
		2	$A_o = \lambda N_o$, then $\lambda = \frac{3630}{9.466 \times 10^{14}} = 3.835 \times 10^{-12}\ s^{-1}$ $T = \frac{\ln 2}{\lambda} = \frac{\ln 2}{3.835 \times 10^{-12}} = 1.807 \times 10^{11}$ s = **5730 y**	**1**
	6		$E_{lib} = \Delta mc^2 = (13.99995) - (13.99923 + 5.486 \times 10^{-4})] \times 931.5 \frac{MeV}{c^2} \times c^2$ E_{lib} = **0.1596591 MeV** = $0.1596591 \times 1.6 \times 10^{-13}$ J = **2.55**	**1** **0.25**
	7		$E' = E_{lib} \times N_{decay} = E_{lib} \times A \times t = 2.55 \times 10^{-14} \times 3650 \times 3600 \times 24 \times 365$ Therefore, $\mathbf{E' = 2.94 \times 10^{-3}}$ **J**	**0.75**
2	1		Antineutrino does not interact with matter	**0.25**
	1		$ED_1 = \frac{E'}{m} \times QF = \frac{2.94 \times 10^{-3}}{70} \times 1 = 4.2 \times 10^{-5}$ Sv = 0.042 mSv $ED = ED_1 + ED_2 = 0.042 + 0.268 = 0.31$ mSv	**0.75**
	2		0.31 mSv ≪ 5 mSv Therefore, **it is safe** for this person form himself radiation	**0.25**