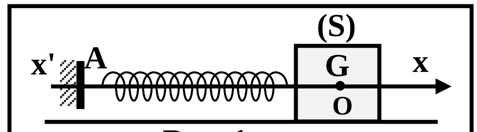
مسابقة في مادة الفيزياء
المدة: ثلاث ساعات

الاسم:
الرقم:

This exam is formed of four obligatory exercises in four pages.
The use of non-programmable calculators is recommended.

Exercise 1 (8 points) Free damped mechanical oscillations

Consider a mechanical oscillator formed by a rigid object (S) of mass m and a horizontal spring of spring constant k and of negligible mass. (S) is attached to one end of the spring, and the other end is fixed to a support A. (S) may move on a horizontal surface with its center of mass G being on a horizontal x-axis (Doc. 1)



Doc. 1

At equilibrium, G coincides with the origin O of the x-axis. (S) is shifted horizontally in the positive direction from its equilibrium position. At the instant $t_0 = 0$, the abscissa of G is X_m and (S) is released without initial velocity.

At an instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic value of its velocity $v = x' = \frac{dx}{dt}$.

During its motion, (S) is subjected to several forces including the tension force $\vec{F} = -k\,x\,\vec{i}$ of the spring and the friction force $\vec{f} = -h\,\vec{v}$, where h is a positive constant called the damping coefficient.

Take the horizontal plane containing G as a reference level for gravitational potential energy.

The aim of this exercise is to study the effect of friction on the oscillations and to determine the value of h.

1) Theoretical study

1-1) Show that: $m\frac{dv}{dt} + k\,x = -hv$ by applying Newton's second law $\sum \vec{F}_{ext} = m\frac{d\vec{v}}{dt}$.

1-2) Write the expression of the mechanical energy ME of the system (Oscillator - Earth) at an instant t in terms of m, k, x and v.

1-3) Deduce that $\frac{dME}{dt} = -hv^2$.

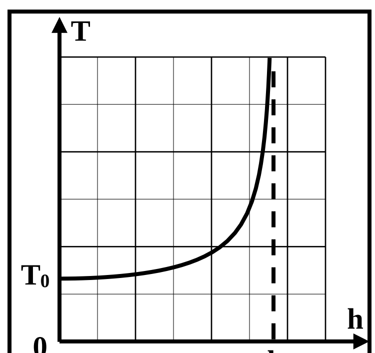
1-4) Establish the second order differential equation that governs the variation of x.

1-5) The center of mass G oscillates with an angular frequency

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{h}{2m}\right)^2}$$

. Deduce the expression of the pseudo-period T.

1-6) For different values of h, we obtain the curve of document 2 which represents T as a function of h, for $0 \le h < h_0$.



Doc. 2

1-6-1) How does T vary for $0 \le h < h_0$?

1-6-2) T_0 represents the proper period of oscillation of G. Justify by referring to document 2.

1-7) Deduce the expression of T_0 in terms of m and k.

2) Experimental study

In the experimental study, we take m = 0.5 kg and k = 100 N/m.

2-1) Calculate the value of T_0.

2-2) The curve of document 3 represents x as a function of time t. Use document 3 to:

2-2-1) determine the pseudo-period T;

2-2-2) give two indicators showing that (S) is submitted to a friction force.

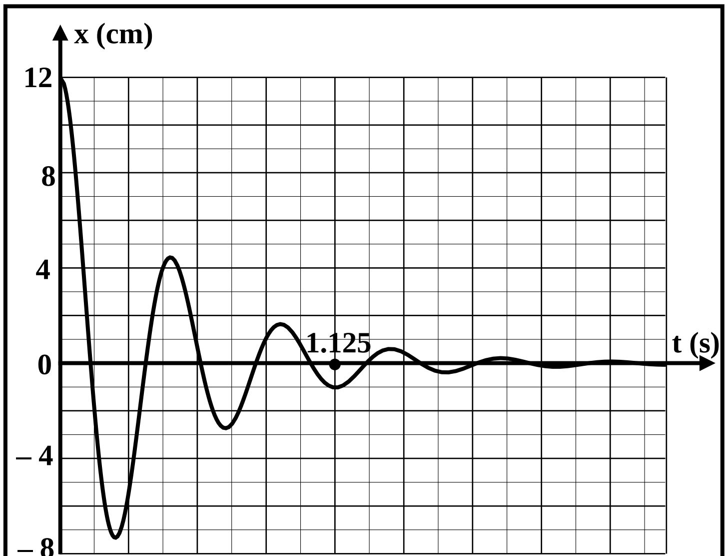
2-3) Calculate h.

2-4) In order to determine again the value of h, an appropriate device is used to trace the curves of ME and the kinetic energy KE of (S) as functions of time, and also the tangent to the curve of ME at t = 0.27 s (Doc. 4).

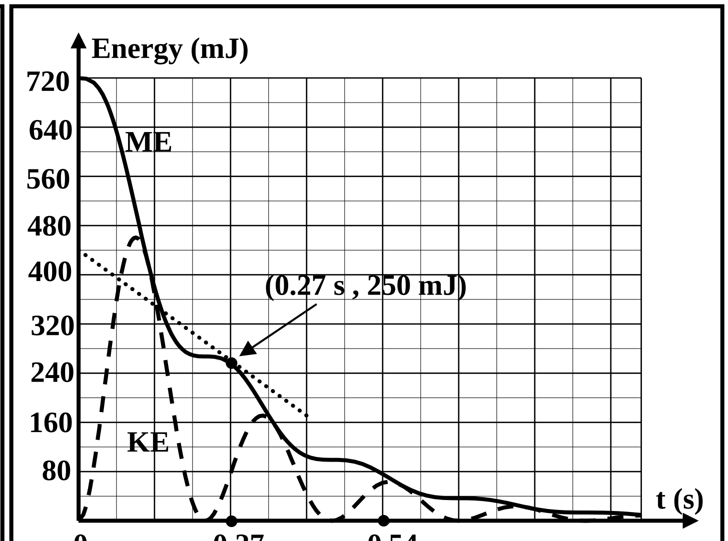
2-4-1) Determine the speed of G at t = 0.27 s by using the curve of KE.

2-4-2) Determine $\frac{dME}{dt}$ at t = 0.27 s.

2-4-3) Deduce again the value of h.



Doc. 3



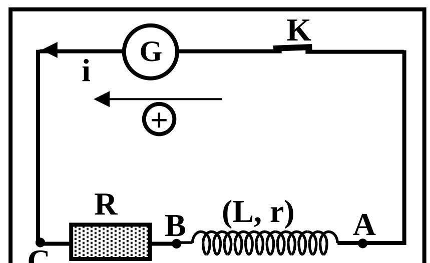
Doc. 4

Exercise 2 (8 points) **Characteristics of a coil**

The aim of this exercise is to determine the characteristics of a coil by two methods. We connect in series a generator (G), a switch K, a resistor of resistance R = 90 Ω and a coil of inductance L and resistance r (Doc. 5).

We close the switch K at the instant $t_0 = 0$.

At an instant t, the circuit carries a current i.



Doc. 5

1) First method

(G) is a generator providing a constant voltage $u_{CA} = E$.

An appropriate device traces the curves of $u_{CB} = u_R$ and $u_{BA} = u_{coil}$ as functions of time (Doc. 6).

1-1) Using the curves of document 6:

1-1-1) determine the value of E;

1-1-2) determine the value of the current I_0 at the steady state;

1-1-3) show that r = 10 Ω.

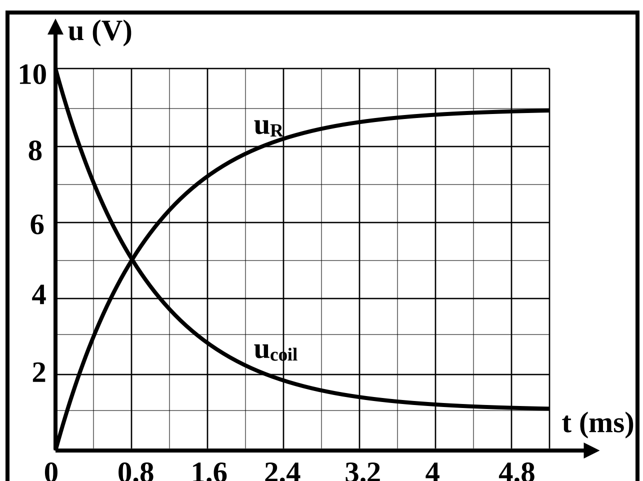
1-2) Establish the first order differential equation in i by applying the law of addition of voltages.

1-3) The solution of this differential equation is

$i = I_0\,(1 - e^{\frac{-(R+r)}{L}t})$. Deduce the expressions of u_R and u_{coil} in terms of R, r, L, I_0 and t.

1-4) $u_{coil} = u_R$ at an instant t_1.

Show that $t_1 = -\frac{L}{R+r}\ln\left(\frac{R-r}{2R}\right)$.



Doc. 6

1-5) Deduce the value of L using document 6.

2) Second method

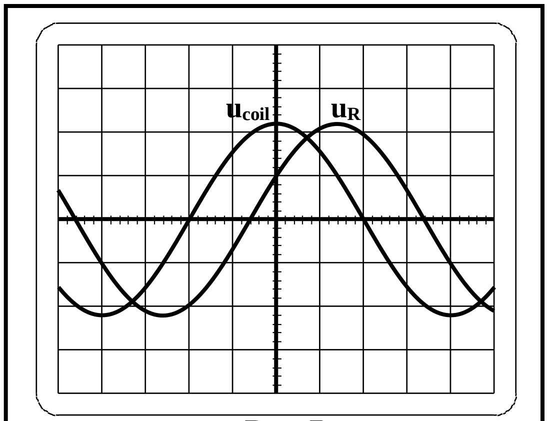
The generator (G) provides now an alternating sinusoidal voltage of angular frequency ω.

An oscilloscope is connected conveniently in the circuit in order to display $u_{CB} = u_R$ on channel 1 and $u_{BA} = u_{coil}$ on channel 2 (Doc. 7).

The adjustments of the oscilloscope:

Horizontal sensitivity: $S_h = 4$ ms/div

Vertical sensitivity: For Ch_1: $S_{V1} = 4$ V/div; For Ch_2: $S_{V2} = 1$ V/div



Doc. 7

2-1) The circuit carries an alternating sinusoidal current $i = I_m \sin(\omega t)$, (SI). Determine the expression of u_{coil} in terms of L, ω, I_m , r and t.

2-2) The expression of the voltage across the coil is of the form: $u_{coil} = A \sin(\omega t) + B \cos(\omega t)$ where A and B are constants. Determine A and B in terms of r, L, I_m and ω.

2-3) Use document 7 to calculate:

2-3-1) the values of I_m and ω;

2-3-2) the maximum voltage U_m across the coil;

2-3-3) the phase difference φ between u_{coil} and u_R.

2-4) Determine again the values of L and r knowing that $\tan\varphi = \dfrac{L\omega}{r}$ and $U_m^2 = A^2 + B^2$.

Exercise 3 (7 points) **Decay of radon-219**

The aim of this exercise is to determine the values of the power and the energy of the electromagnetic radiation γ emitted in the disintegration of radon-219.

The radionuclide radon $^{219}_{86}Rn$ decays into polonium $^{A}_{Z}Po$ with the emission of an α particle and γ radiation of energy E_γ according to the following equation: $^{219}_{86}Rn \rightarrow {}^{A}_{Z}Po + \alpha + \gamma$

Given: $m(^{219}_{86}Rn) = 204007.3316$ MeV/c^2 ; $m(^{A}_{Z}Po) = 200271.9597$ MeV/c^2 ; $m(\alpha) = 3728.4219$ MeV/c^2

1 MeV = 1.602×10^{-13} J ; Molar mass of $^{219}_{86}Rn$ is 219 g/mol ; $N_A = 6.022\times10^{23}$ mol^{-1}

1) Calculate A and Z, indicating the used laws.
2) Calculate the energy (in MeV) liberated by the decay of one nucleus of radon-219.
3) Deduce that the energy of the emitted γ radiation is $E_\gamma = 0.195$ MeV knowing that the radon nucleus is at rest, the kinetic energy of the emitted α particle is 6.755 MeV and the kinetic energy of the polonium nucleus is negligible.
4) The initial mass of a radon sample is $m_0 = 8$ g at $t_0 = 0$. Show that the initial number N_0 of radon nuclei present in the sample at $t_0 = 0$ is $N_0 = 21.998 \times10^{21}$ nuclei.
5) Calculate the number of the α particles emitted between $t_0 = 0$ and $t_1 = 10$ s, knowing that the remaining number of radon nuclei at $t_1 = 10$ s is $N = 3.998 \times10^{21}$ nuclei.
6) Calculate the values of the decay constant λ and the half-life T of radon-219.
7) Calculate, in becquerel, the activity A_1 of the radon sample at the instant $t_1 = 10$ s.
8) The energy of the emitted γ radiation between the instant $t_0 = 0$ and an instant t is $E = N_d E_\gamma$ where N_d is the number of the decayed nuclei of radon-219 between these two instants.

   8-1) Show that $E = N_0 E_\gamma (1 - e^{-\lambda t})$.

   8-2) Deduce the value of E during the time interval $[0, \infty[$.

9) The power p of the emitted γ radiation at an instant t is given by: $p = \frac{dE}{dt}$.

9-1) Show that $p = \lambda N_0 E_\gamma e^{-\lambda t}$.

9-2) Deduce the maximum power P_{max} of the γ radiation.

9-3) Deduce the power of the γ radiation as $t \to \infty$.

Exercise 4 (7 points) — Interference of light

The aim of this exercise is to study the phenomenon of interference of light using Young's double-slit set-up.

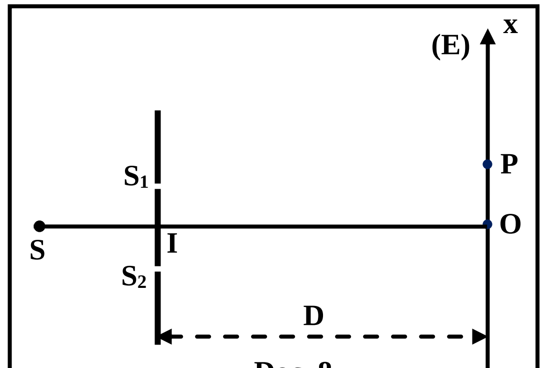
Document 8 shows Young's double-slit set-up, which is constituted of two thin parallel and horizontal slits S_1 and S_2 separated by a distance a = 0.5 mm, and a screen (E) placed parallel to the plane of the two slits at a distance D = 2 m.

A point source S, equidistant from S_1 and S_2, illuminates the two slits by monochromatic radiation of wavelength λ = 600 nm in air.

(OI) is the perpendicular bisector of the segment $[S_1S_2]$.

The expression of the optical path difference at point P on the vertical x-axis in the interference pattern is:

$\delta = (SS_2 + S_2P) - (SS_1 + S_1P) = \frac{a\,x}{D}$ where $x = \overline{OP}$.



Doc. 8

1) Describe the interference pattern on the screen (E).

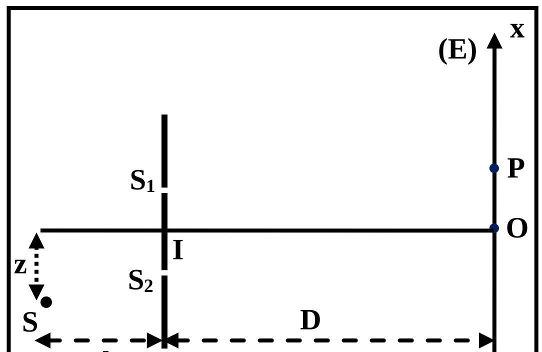
2) Show that O is the center of the central bright fringe.

3) Suppose that P is the center of a dark fringe of order k ($k \in \mathbb{Z}$).

3-1) Give the expression of the optical path difference δ at point P in terms of k and λ.

3-2) Deduce the expression of the abscissa x_k of P in terms of k, λ, D and a.

3-3) Determine the order of the dark fringe at P knowing that x_k = 6 mm.



Doc. 9

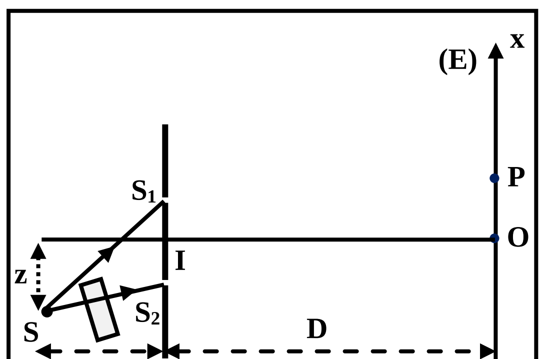
4) The point source S which is placed at a distance d from the plane of the slits, is moved by a displacement z, in the negative direction, to the side of S_2 parallel to the x-axis (Doc. 9).

The optical path difference at point P becomes: $\delta = \frac{a\,z}{d} + \frac{a\,x}{D}$.

4-1) Determine the position of the center O' of the central bright fringe in terms of D, z and d.

4-2) Specify whether the central bright fringe is displaced to the side of S_1 or to the side S_2.

4-3) A thin transparent plate of parallel faces, of thickness e = 0.02 mm and of refractive index n = 1.5, is placed in front of S_2 (Doc.10).



Doc. 10

The optical path difference at point P becomes:

$$\delta = \frac{a\,z}{d} + \frac{a\,x}{D} + e\,(n-1).$$

We adjust the distance d in order that the center of the central bright fringe returns back to the point O. Determine the value of d knowing that $|z|$ = 0.4 cm.

أسس التصحيح – مادة الفيزياء - إنكليزي

Exercise 1 (8 points) **Free damped mechanical oscillations**

Part			Answer	Mark
1	1-1		$m\vec{g} + \vec{N} + \vec{f} + \vec{T} = m\dfrac{d\vec{v}}{dt}$; projecting the vectors along the x-axis $0 + 0 + – hv – k\,x = m\dfrac{dv}{dt}$, thus $m\dfrac{dv}{dt} + k\,x = – hv$	0.75
	1-2		$ME = ½\, m\, v^2 + ½\, k\, x^2$	0.25
	1-3		$\dfrac{dME}{dt} = m\,v\dfrac{dv}{dt} + k\,x\dfrac{dx}{dt} = v\,(m\dfrac{dv}{dt} + k\,x)$; substituting $m\dfrac{dv}{dt} + k\,x = – hv$ gives $\dfrac{dME}{dt} = v\,(– hv)$, thus $\dfrac{dEM}{dt} = – h\,v^2$	0.5
	1-4		$\dfrac{dME}{dt} = v\,(m\dfrac{dv}{dt} + k\,x) = – h\,v^2$, then $m\,x'' + h\,x' + k\,x = 0$	0.5
	1-5		$T = \dfrac{2\,\pi}{\omega} = \dfrac{2\,\pi}{\sqrt{\frac{k}{m} - \left(\frac{h}{2m}\right)^2}}$	0.25
	1-6	1	As h increases T increases	0.25
		2	Graphically h = 0 , for $T = T_0$ therefore T_0 is the proper period	0.25
	1-7		For h = 0 , $T = T_o = \dfrac{2\,\pi}{\sqrt{\frac{k}{m} - 0}}$ so $T_o = 2\,\pi\sqrt{\dfrac{m}{k}}$	0.5
2	2-1		$T_o = 2\,\pi\sqrt{\dfrac{m}{k}} = 2 \times 3.14\sqrt{\dfrac{0.5}{100}} = 0.444$ s	0.5
	2-2	1	2.5 T = 1.125 s , thus T = 0.45 s.	0.5
		2	X_m decreases with time and T is greater than $T_o (T > T_o)$	0.5
	2-3		$T = \dfrac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{h}{2m}\right)^2}}$, so $\dfrac{k}{m} - \dfrac{h^2}{4m^2} = \dfrac{4\,\pi^2}{T^2}$, then $h^2 = 4mk - \dfrac{16\,m^2\pi^2}{T^2}$ so $h=\sqrt{4mk - \dfrac{16\,m^2\pi^2}{T^2}}$ $= \sqrt{4(0.5)(100) - \dfrac{16(0.5^2)(\pi^2)}{0.45^2}} = 2.24$ kg/s	1
	2-4	1	For t = 0.27 s, KE = 80 mJ ,alors $½\, m\, V^2 = 0.08$ et $V = \sqrt{\dfrac{2 \times 0.08}{0.5}} = 0.566$ m/s.	0.75
		2	$\dfrac{dEM}{dt} = \dfrac{\Delta EM}{\Delta t} =$ slope $= \dfrac{0.25 - 0.440}{0.27 - 0} = – 0.704$ J/s	0.75
		3	$\dfrac{dEM}{dt} = -0.704 = –h\,V^2$, thus $h = \dfrac{0.704}{0.566^2} = 2.2$ kg/s	0.75

Exercise 2 (8 points) Characteristics of a coil

Part			Answer	Mark
1	1-1	1	Law of addition of voltages: $u_{CA} = u_{CB} + u_{BA}$ At steady state $u_{CB} = u_R = 9V$ and $u_{BA} = u_{coil} = 1V$; thus $u_{CA} = E = 9+1 = 10$ V **Or** : $E = u_{coil} + u_R$. At t = 0, i = 0 so $u_R = 0$ then $E = u_{coil\ (0)} = 10V$	0.75
		2	At steady state $u_{CB} = u_R = 9 = R \times I_0$ thus $9 = 90 \times I_0$; $I_0 = 0.1$ A	0.5
		3	$u_{BA} = u_{coil} = ri + L\frac{di}{dt}$; at steady state $u_{BA} = 1V$ and $\frac{di}{dt} = 0$ so $1 = r\,I_0$; $r = \frac{1}{0.1} = 10\ \Omega$ **Or** : At the steady state $I_0 = E/(R+r)$, then r = 10 Ω	0.5
	1-2		Law of addition of voltages : $u_{CA} = u_{CB} + u_{BA}$; $E = Ri + ri + L\frac{di}{dt} = (R+r)i + L\frac{di}{dt}$	0.5
	1-3		$U_{coil} = ri + L\frac{di}{dt} = r\,I_0(1 - e^{\frac{-(R+r)}{L}t}) + L(R+r)\frac{I_0}{L}e^{\frac{-(R+r)}{L}t} = r\,I_0 + RI_0\,e^{\frac{-(R+r)}{L}t}$ $u_R = Ri = RI_0(1 - e^{\frac{-(R+r)}{L}t})$	0.5 0.25
	1-4		$u_{coil} = u_R$; $r\,I_0 + RI_0\,e^{\frac{-(R+r)}{L}t} = RI_0\left(1 - e^{\frac{-(R+r)}{L}t}\right)$; $(R - r)\,I_0 = 2\,RI_0\,e^{\frac{-(R+r)}{L}t}$ then $e^{\frac{-(R+r)}{L}t} = \frac{R-r}{2R}$ so $-\frac{(R+r)t1}{L} = \ln(\frac{R-r}{2R})$ $t_1 = -\frac{L}{R+r} \times \ell n\left(\frac{R-r}{2R}\right)$	0.75
	1-5		$L = \frac{-(R+r)\times t1}{\ell n\left(\frac{R-r}{2R}\right)} = \frac{-(90+10)\times 0.0008}{\ell n(\frac{90-10}{180})} = 0.099$ H	0.75
2	2-1		$u_{coil} = ri + L\frac{di}{dt} = rI_m\sin(\omega t) + L\,\omega\,I_m\cos(\omega t)$	0.5
	2-2		$U_{coil} = rI_m\sin(\omega t) + L\,\omega\,I_m\cos(\omega t) = A\sin(\omega t) + B\cos(\omega t)$ therefore $A = rI_m$ and $B = L\,\omega\,I_m$	0.5
	2-3	1	$U_{Rm} = 4V/div \times 2.2div = 8.8$ V ; $I_m = \frac{URm}{R} = \frac{8{,}8}{90} = 0.097A$ $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{8\times4\times10^{-3}} = 62.5\pi\ rad/s = 196.35 rad/s$	0.5 0.5
		2	$U_m = 2.2div \times 1V/div = 2.2V$	0.25
		3	$\varphi = \frac{2\pi\times1{,}4div}{8div} = \frac{7\pi}{20}$ rad =1.099 rad	0.5
	2-4		$\tan\varphi = \frac{L\omega}{r}$; $\tan 1.099 = \frac{L\times62.5\pi}{r}$ donc $\frac{L}{r} = \frac{\tan 1{,}099}{62.5\pi}$; $L = \frac{\tan 1.099}{62.5\pi}\times r$. ① $U_m^2 = A^2 + B^2$; $9 = (rI_m)^2 + (L\,\omega I_m)^2$ ② By replacing ① in ② : $2.2^2 = r^2 \times I_m^2\,(1 + \omega^2 \times \left(\frac{\tan 1.099}{62.5\pi}\right)^2)$; Therfore r = 9.998 Ω and L = 0.0998 H **or**: $U_m^2 = A^2 + B^2$ then $U_m^2 = (L\omega)^2 I_{max}^2 + r^2 I_{max}^2$ but $\tan\varphi = \frac{L\omega}{r}$; $L\,\omega = r\tan\varphi$; Then $U_m^2 = r^2 I_{max}^2\,(1 + tan^2\varphi)$ we obtain : $r = \frac{U_m}{I_m\sqrt{1+(tan\varphi)^2}} = 10{,}3\ \Omega$ so L = 0,1 H	0.75

Exercise 3 (7 points) **Decay of radon-219**

Part		Answer	Mark
1		Law of conservation of mass number A : 219 = A + 4 + 0 , so A = 215. Law of conservation of charge number Z : 86 = Z + 2 + 0 so Z = 84.	**1**
2		$E_{lib} = \Delta m \times c^2 = [(m\ {}^{219}_{86}Rn)$ -(m ${}^{A}_{Z}Po$ + mα)]c^2 E_{lib} == 204007.3316 – (200271.9597 + 3728.4219) = 6.95MeV	**0.75**
3		$E_{lib} = KE_{(\alpha)} + E_\gamma$, then E_γ = 6.95 – 6.755 , then E_γ = 0.195MeV	**0.5**
4		$N_o = \frac{m_0}{M} N_A = \frac{8}{219} \times 6.022 \times 10^{23} = 21.998 \times 10^{21}$ noyaux	**0.5**
5		$N_\alpha = N_d = N_o - N = 21.998 \times 10^{21} - 3.998 \times 10^{21} = 18 \times 10^{21}$ nuclei	**0.5**
6		$N = N_o e^{-\lambda t}$, so $\lambda t = -\ln\frac{N}{N_o} = -\ln\left(\frac{3.998 \times 10^{21}}{21.998 \times 10^{21}}\right)$ then $\lambda = \frac{-1}{10} \times \ln\left(\frac{3.998 \times 10^{21}}{21.998 \times 10^{21}}\right) = 0.1705\ s^{-1}$ $T = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} = \frac{0.693}{0.1705} = 4.06$ s	**1**
7		$A = \lambda N = 0.1705 \times 3.998 \times 10^{21} = 68.1659 \times 10^{19}$ Bq	**0.5**
8	8-1	$E = N_d E\gamma = (N_o - N)E\gamma = \left(N_o - N_o\ e^{-\lambda t}\right)E\gamma$, so $E = N_o\ E\gamma\ (1 - e^{-\lambda t})$	**0.25**
	8-2	For t → ∞, $E = N_o\ E_\gamma(1-0) = 21.998 \times 10^{21} \times 0.195 \times 1.602 \times 10^{-13}$ Then $E = 6.87 \times 10^8$ J	**0.5**
9	9-1	$P = \frac{dE}{dt} = \frac{d(N_o\ E_\gamma\ (1 - e^{-\lambda t}))}{dt} = \lambda\ N_o\ E_\gamma\ \ e^{-\lambda t}$	**0.5**
	9-2	For t = 0 ; $p = p_{max} = \lambda\ N_o\ E_\gamma\ \ e^{-\lambda(0)} = \lambda\ N_o\ E_\gamma$ $p = 0.1705 \times 21.998 \times 10^{21} \times 0.195 \times 1.602 \times 10^{-13} = 11.72 \times 10^7$ W	**0.75**
	9-3	For t →∞, $P_\infty = \lambda\ N_o\ E_\gamma\ \ e^{-\lambda(\infty)} = 0$	**0.25**

Exercise 4 (7 points) **Interference of light**

Part		Answer	Mark
1		We observe alternate bright and dark fringes which are rectilinear, equidistant and parallel to the slits	1
2		$x_O = 0$ then $\delta_o = \frac{a\,x}{D} = 0$	0.5
3	3-1	$\delta = (2K+1)\frac{\lambda}{2}$ with $k \in Z$	0.5
	3-2	$(2K+1)\frac{\lambda}{2} = \frac{a\,x}{D}$, then $x_k = (2k+1)\frac{\lambda D}{2a}$ with $k \in Z$	1
	3-3	$x_k = (2k+1)\frac{\lambda D}{2a}$ then $6 \times 10^{-3} = (2k+1)\frac{600\times10^{-9}\times2}{2\times0.5\times10^{-3}}$, we obtain k = 2	1
4	4-1	$\delta_{O'} = \frac{az}{d} + \frac{ax}{D} = 0$, so $x_{O'} = \frac{-zD}{d}$	1
	4-2	Z < 0 and D > 0 ; d >0 then $x_{O'} > 0$, therefore the central bright fringe is displaced to the side of S_1	0.75
	4-3	$\delta_O = 0$ and $x_O = 0$, but $\delta = \frac{az}{d} + \frac{ax}{D} + e(n-1)$ $d = \frac{-a\,Z}{e\,(n-1)} = \frac{-(0.5\times10^{-3})(-0.4\times10^{-2})}{(0.02\times10^{-3})(1.5-1)}$, so d = 0.2 m	1.25