

الاسم:  
الرقم:

مسابقة في مادة الفيزياء  
المدة ثلاث ساعات

**This exam is formed of four exercises in four pages numbered from 1 to 4.  
The use of non-programmable calculator is recommended,**

**First exercise: (7.5 points) Variation of the kinetic energy of a system**

The aim of this exercise is to verify the theorem of kinetic energy of a system.

The skier (S) of mass  $M = 80 \text{ kg}$ , moves down from O to A, with a constant velocity  $\vec{v} = v\vec{i}$ , where  $v = 30 \text{ m/s}$  along the line of greatest slope of a track inclined by an angle  $\alpha = 30^\circ$  with the horizontal. The track exerts on the skier a constant force of friction  $\vec{f} = -f\vec{i}$ .

The motion of the skier is represented by the motion of its center of mass G on  $\vec{x}'x$  where  $\vec{i}$  is a unit vector along this axis (figure 1).

Neglect the air resistance on the skier.

Take:

- the horizontal plane through B as a gravitational potential energy reference for the system (skier, Earth).
- $g = 10 \text{ m/s}^2$ .

1) Name and represent the external forces acting on G along the path OA.

2) a) Show that the linear momentum  $\vec{P}$  of the skier is constant.

b) Apply Newton's second law on the skier, between the points O and A, deduce the magnitude of  $\vec{f}$ .

3) The skier, upon reaching A, starts exerting a constant braking force  $\vec{f}_1 = -f_1\vec{i}$  to stop at B. The skier covers the distance AB during a time interval  $\Delta t = 3 \text{ s}$ .

a) Determine the magnitude of  $\vec{f}_1$ , assuming that  $\frac{\Delta \vec{P}}{\Delta t} \approx \frac{d\vec{P}}{dt}$ .

b) The mechanical energy of the system (skier, Earth) decreases from A to B. Name the forces that are responsible of this decrease.

c) Determine the distance AB covered by the skier during the time interval  $\Delta t$ .

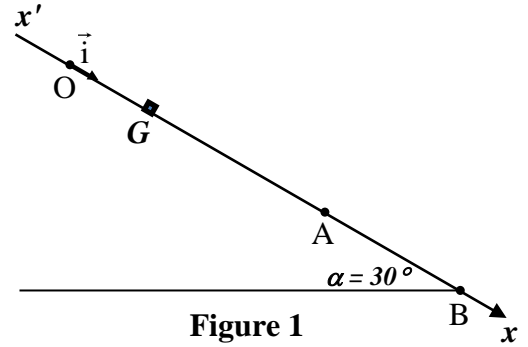
4) a) Determine between A and B :

- the variation of the gravitational potential energy  $\Delta PE_g$  of the system (skier, Earth) ;
- the work done by the weight  $W_{mg}$ .

b) Compare  $\Delta PE_g$  and  $W_{mg}$ .

5)  $\Delta KE$  and  $\sum W_{\vec{F}_{ext}}$  are respectively the variation of the kinetic energy of the skier and the algebraic sum of the work done by the external forces between A and B.

Verify, between A and B, the work-energy theorem:  $\Delta KE = \sum W_{\vec{F}_{ext}}$ .



## Second exercise: (7.5 points) The characteristics of RLC series circuit

Consider:

- a generator G delivering an alternating sinusoidal voltage :  
 $u_{AM} = u_G = u = U\sqrt{2} \cos \omega t$  (u in V and t in s), where  $U = 5$  V and  $\omega = 2\pi f$  with adjustable frequency f;
- a coil of inductance L and of negligible resistance;
- a capacitor of capacitance C;
- a resistor of resistance  $R = 150 \Omega$ ;
- an oscilloscope;
- a milli-ammeter of negligible resistance;
- a switch K and connecting wires.

In order to determine L and C, we perform the following experiments:

### A- First experiment

We perform successively the setup of figure 1 and of figure 2.

For  $f = 500$  Hz, the effective current I, indicated by the milli-ammeter, has the same value  $I = 50$  mA in both setups. Take  $\frac{1}{\pi} = 0.32$ .

- 1) The coil is connected across the terminals of G (figure 1). The circuit carries a current i of expression  $I = I\sqrt{2} \cos(\omega t - \frac{\pi}{2})$ . (i in A and t in s)
  - a) Determine the expression of the voltage  $u_{BD} = u_{\text{coil}}$  in terms of L,  $\omega$ , I and t.
  - b) Deduce the value of L.
- 2) The capacitor is connected across the terminals of G (figure 2). The circuit carries a current i of expression  $i = I\sqrt{2} \cos(\omega t + \frac{\pi}{2})$ .
  - a) Determine the expression of the voltage  $u_{BD} = u_C$  in terms of C,  $\omega$ , I and t.
  - b) Deduce the value of C.

### B- Second experiment

To verify the values obtained for L and C in the first experiment, we perform the setup of the circuit shown in Figure 3. This circuit contains the generator, the coil, the capacitor, and the resistor of resistance  $R = 150 \Omega$ . The oscilloscope, displays on channel (1), the voltage  $u_{AM}$  across the generator, and on channel (2), the voltage  $u_{DM}$  across the resistor. Figure (4) shows the waveforms representing  $u_{AM}$  and  $u_{DM}$ .

The circuit carries a current  $i = I\sqrt{2} \cos(\omega t + \varphi)$ .

- 1) Redraw figure 3 and indicate the connections of the oscilloscope.
- 2) Apply the law of addition of voltages and give t a particular

value, show that:  $\tan \varphi = \frac{\frac{1}{C\omega} - L\omega}{R}$ .

- 3) Referring to the waveform of Figure 4 observed on the screen of the oscilloscope, determine:
  - a) the frequency f;
  - b) the phase difference  $\varphi$  between u and i.
- 4) The effective voltage U being kept constant and we vary f. We observe that  $u_{AM}$  and  $u_{DM}$  become in phase when f takes the value  $f_0 = 500$  Hz.
  - a) Name the phenomenon that takes place.
  - b) Give the relation giving  $\omega_0$  in terms of L and C.
- 5) Determine L and C.

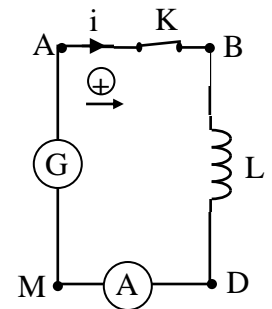


Figure 1

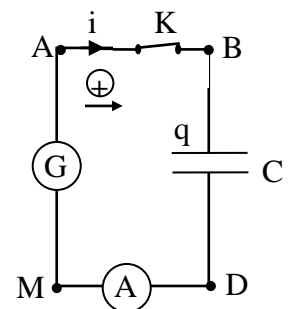


Figure 2

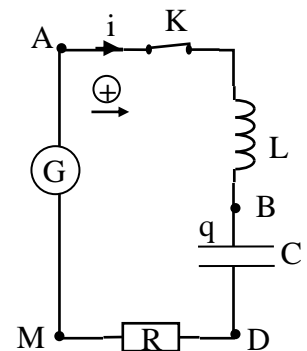


Figure 3

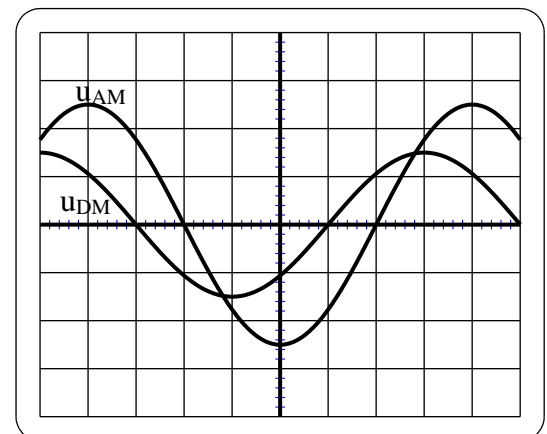


Figure 4

Horizontal Sensitivity : 0.5 ms/div

### Third exercise: (7.5 points)      **Corpuscular aspect of light**

The aim of this exercise is to study the emission spectrum of the hydrogen atom and use the emitted light to produce photoelectric effect.

**Given:**

- Planck's constant:  $h = 6.62 \times 10^{-34} \text{ J}\cdot\text{s}$  ;
- Speed of light in vacuum:  $c = 3 \times 10^8 \text{ m/s}$  ;
- $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$  ;
- Elementary charge:  $e = 1.602 \times 10^{-19} \text{ C}$  ;
- $1 \text{ nm} = 10^{-9} \text{ m}$ .

#### **A. Hydrogen atom**

The emission spectrum of the hydrogen atom constituted in its visible part of four radiations denoted by  $H_\alpha$ ,  $H_\beta$ ,  $H_\gamma$  and  $H_\delta$  of respective wavelengths, in vacuum, 656.27nm, 486.13nm, 435.05nm and 410.17nm.

**I.** In 1885, Balmer noticed that the wavelengths  $\lambda$  of these four radiations verify the empirical formula

$$\lambda = \lambda_0 \frac{n^2}{n^2 - 4} \text{ where } \lambda_0 = 364.6 \text{ nm where } n \text{ is a non-zero positive whole number.}$$

- 1) The smallest value of  $n$  is 3. Justify.
- 2) Calculate the wavelength corresponding to this radiation.
- 3) Deduce the values of  $n$  corresponding to the wavelengths of the other three visible radiations in the emission spectrum of the hydrogen atom.

**II.** The quantized energy levels of the hydrogen atom are given by the formula:

$$E_n = - \frac{13.6}{n^2} \text{ (in eV) where } n \text{ is a whole non-zero positive number.}$$

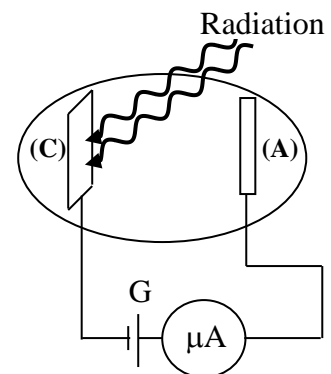
Using the expression of  $E_n$ , determine the energy of the atom when it is:

- 1) in the ground state.
- 2) in each of the first five excited levels.
- 3) ionized state.

#### **B. Photoelectric effect**

A hydrogen lamp of power  $P_S = 2\text{W}$ , emits uniformly radiation in all directions in a homogeneous and non-absorbing medium. This lamp illuminates a potassium cathode C of a photoelectric cell of work function  $W_0 = 2.20 \text{ eV}$  and of a surface area  $s = 2\text{cm}^2$  placed at a distance  $D = 1.25\text{m}$  from the lamp (figure1).

- 1) Calculate the threshold wavelength of the potassium cathode.
- 2) Among the rays of Balmer series, specify the radiation that can produce photoelectric emission.
- 3) Using a filter we illuminate the cell by a blue light  $H_\beta$  of wavelength  $\lambda = 486.13\text{nm}$ . The generator G is adjusted so that the anode (A) captures all the emitted electrons by the cathode of quantum efficiency  $r = 0.875\%$ .
  - a) Show that the received power of the radiation  $P_0$  of the cell is  $2.04 \times 10^{-5}\text{W}$ .
  - b) Determine the number  $N_0$  of the incident photons received by the cathode C in one second.
  - c) Determine the current in the circuit.



**Figure 1**

### Fourth exercise: (7.5 points)

### Compound Pendulum

The aim of this exercise is to study the motion of a compound pendulum.

Consider a compound pendulum (P) consists:

- of a straight and homogeneous rod (R) of length  $AB = \ell$  and of mass  $m$  ;
- of a solid (S), taken as a particle of mass  $m_1$ , free to slide along the part OB of the rod, O being the midpoint of the rod.

We fix (S) at a point C such that  $\overline{OC} = x$  ( $x > 0$ ).

(P) can oscillate, in a vertical plane, around a horizontal axis ( $\Delta$ ) perpendicular to the rod at O ( figure 1).

(P) is shifted from its equilibrium position by a small angle  $\theta_m$  then released without initial velocity at the instant  $t_0 = 0$ , the pendulum oscillate then, without friction, around its equilibrium position.

At the instant  $t$ , the angular elongation of the pendulum is  $\theta$  and its angular velocity is  $\theta' = \frac{d\theta}{dt}$ .

Given: moment of inertia of the rod about the axis of rotation ( $\Delta$ ):  $I_0 = \frac{1}{12} m \ell^2$ ,  $m = 3m_1$ ,

$\ell = 0.5$  m,  $g = 10$  m/s<sup>2</sup> and  $\pi^2 = 10$ .

For small  $\theta$ :  $\cos \theta \approx 1 - \frac{\theta^2}{2}$  and  $\sin \theta \approx \theta$  ( $\theta$  in rd).

G is the center of inertia of the pendulum and the horizontal plane passing through O is taken as reference level of the gravitational potential energy.

1) Show that:

a)  $\overline{OG} = \frac{x}{4}$ ;

b) The expression of the moment of inertia of the pendulum is:  $I = \frac{m}{12} (\ell^2 + 4x^2)$ .

2) Determine the expression the mechanical energy of the system (pendulum, Earth) in terms of  $\theta$ ,  $\theta'$ ,  $m$ ,  $x$  and  $\ell$ .

3) a) Establish the second order differential equation in  $\theta$  which governs the oscillations of the pendulum.

b) Deduce that the expression of the proper period of the pendulum is:  $T_0 = \sqrt{\frac{4x^2 + \ell^2}{x}}$ .

4) a) Determine the value of  $x$  for which  $T_0$  is minimum.

b) Deduce that  $T_{0(\min)} = 1.41$  s.

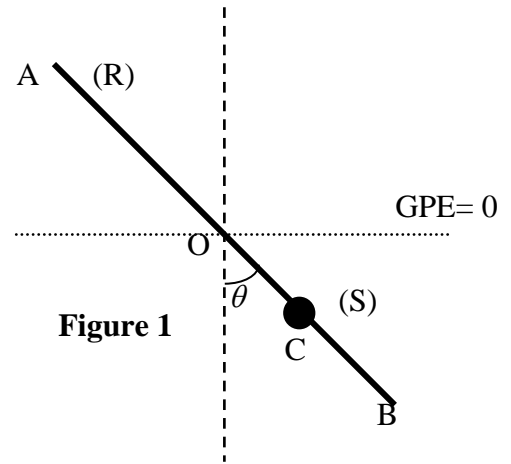
5) Using a coupling device, the pendulum (P) plays the role of an exciter for a simple pendulum ( $P_1$ ) of length  $\ell_1 = 65$  cm. The oscillations of (P) and ( $P_1$ ) are slightly damped.

a) Knowing that the proper period of the simple pendulum, for small oscillation, is  $T = 2\pi\sqrt{\frac{\ell}{g}}$ ,

Calculate the value of the proper period  $T_{01}$  of ( $P_1$ ).

b) i) (P) oscillates now with its minimum period. It is noticed that ( $P_1$ ) does not enter in amplitude resonance with (P). Justify.

ii) We move (S) between O and B. For a value  $x_0$  of  $x$ , we notice that ( $P_1$ ) oscillates with large amplitude. Calculate the value of  $x_0$ .

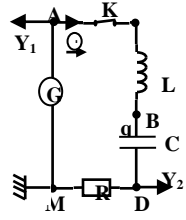


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| دورة العام 2016 العادية<br>الاثنين 13 حزيران 2016 | امتحانات الشهادة الثانوية العامة<br>الفرع : علوم عامة | وزارة التربية والتعليم العالي<br>المديرية العامة للتربية<br>دائرة الامتحانات |
| الاسم:<br>الرقم:                                  | مسابقة في مادة الفيزياء<br>المدة ثلاث ساعات           | مشروع معيار التصحيح  |

### First exercise (7.5 points)

| Part of the Q | Answer   | Mark |
|---------------|--|------|
| 1             | The forces acting on the skier : <ul style="list-style-type: none"> <li>• Normal reaction <math>\vec{N}</math> ;</li> <li>• Weight <math>m\vec{g}</math> ;</li> <li>• The frictional force <math>\vec{f}</math></li> </ul> Diagram.  | 3/4  |
| 2.a           | $\vec{P} = M\vec{V}$ since $\vec{V} = \vec{C}t \Rightarrow \vec{P} = \vec{C}t$ .   | 3/4  |
| 2.b           | $\frac{d\vec{P}}{dt} = \vec{Mg} + \vec{N} + \vec{f} = \vec{0}$ project along x'x: $Mg\sin\alpha - f = 0$<br>$\Rightarrow f = Mg\sin\alpha = 400 \text{ N}$ .   | 1    |
| 3.a           | $\frac{d\vec{P}}{dt} = \vec{Mg} + \vec{f} + \vec{N} + \vec{f}_1 = \frac{\Delta\vec{P}}{\Delta t}$<br>Project along x'x $\Rightarrow -f_1 = \frac{MV_B - MV_A}{\Delta t} = -\frac{MV_A}{\Delta t} \Rightarrow f_1 = 800 \text{ N}$ .<br>Or : $\frac{\Delta\vec{P}}{\Delta t} = \sum \vec{F}_{\text{ext}} \Rightarrow \frac{\vec{P}_O - \vec{P}_A}{\Delta t} = \sum \vec{F}_{\text{ext}}$<br>Project along x'x : $\frac{0 - MV_A}{\Delta t} = Mg\sin\alpha - f - f_1 = 0 - f_1 = -f_1 \Rightarrow f_1 = 800 \text{ N}$ | 1    |
| 3.b           | Because friction and braking forces  | 1/2  |
| 3.c           | $\Delta M.E = W(\vec{f}) + W(\vec{f}_1) \Rightarrow M.E_B - M.E_A = W(\vec{f}) + W(\vec{f}_1) \Rightarrow$<br>$-1/2 MV^2 - Mg AB \sin\alpha = -f \cdot AB - f_1 \cdot AB$<br>$\Rightarrow (40 \times 900) + (400 \times AB) = 1200 \times AB \Rightarrow AB = 45 \text{ m}$ .  | 1    |
| 4.a.i         | $\Delta GPE = GPE_B - GPE_A = 0 - Mg AB \sin\alpha = -Mg AB \sin\alpha = -1800 \text{ J}$  | 3/4  |
| 4.a.ii        | $W(M\vec{g}) = Mgh = Mg AB \sin\alpha = 1800 \text{ J}$  | 1/2  |
| 4.b           | $\Delta(GPE) = -W(M\vec{g})$ .   | 1/4  |
| 5             | $\Delta M.E = \Delta K.E + \Delta GP.E = W(\vec{f}) + W(\vec{f}_1)$<br>$\Rightarrow \Delta K.E = W(M\vec{g}) + W(\vec{f}) + W(\vec{f}_1)$<br>since $W(\vec{N}) = 0 \Rightarrow \Delta K.E = \sum W_{\vec{F}_{\text{ext}}}$<br>Or : $\Delta M.E = \Delta K.E + \Delta GP.E = W(\vec{f}) + W(\vec{f}_1)$<br>$\Rightarrow \Delta K.E = W(\vec{f}) + W(\vec{f}_1) - \Delta GP.E = W(\vec{f}) + W(\vec{f}_1) + W(M\vec{g})$<br>Or $W(\vec{N}) = 0 \Rightarrow \Delta K.E = \sum W_{\vec{F}_{\text{ext}}}$                 | 1    |

## Second exercise (7.5 points)

| Part of the Q | Answer  | Mark  |
|---------------|---|-------|
| A.1.a         | $u_{BD} = u_L = L \frac{di}{dt} = -LI\omega\sqrt{2} \sin(\omega t - \frac{\pi}{2})$   | 3/4   |
| A.1.b         | $u_{AM} = u_{BD} \Rightarrow -LI\omega\sqrt{2} \sin(\omega t - \frac{\pi}{2}) = U\sqrt{2} \cos \omega t \Rightarrow LI\omega\sqrt{2} \cos(\frac{\pi}{2} + \omega t - \frac{\pi}{2}) = U\sqrt{2} \cos \omega t$<br>By comparison: $U\sqrt{2} = LI\omega\sqrt{2} \Rightarrow L = 0.032 \text{ H} = 32 \text{ mH}$ .<br><b>Or:</b> $-LI\omega\sqrt{2} \sin(\omega t - \frac{\pi}{2}) = U\sqrt{2} \cos \omega t$<br>For $t = 0$ : $U\sqrt{2} = LI\omega\sqrt{2} \Rightarrow L = 0.032 \text{ H} = 32 \text{ mH}$ .  | 3/4   |
| A.2.a         | $i = C \frac{du_C}{dt} \Rightarrow u_C = \frac{1}{C} \int i dt = \frac{I\sqrt{2}}{C\omega} \sin(\omega t + \varphi)$  | 3/4   |
| A.2.b         | $u_{AM} = u_{BD} \Rightarrow U\sqrt{2} \cos \omega t = \frac{I\sqrt{2}}{C\omega} \sin(\omega t + \frac{\pi}{2}) \Rightarrow U\sqrt{2} \cos \omega t = \frac{I\sqrt{2}}{C\omega} \cos(\frac{\pi}{2} - \omega t - \frac{\pi}{2})$<br>By comparison: $U\sqrt{2} = \frac{I\sqrt{2}}{C\omega} \Rightarrow C = 3.2 \times 10^{-6} \text{ F} = 3.2 \text{ } \mu\text{F}$<br><b>Or:</b> $u_{AM} = u_{BD} \Rightarrow U\sqrt{2} \cos \omega t = \frac{I\sqrt{2}}{C\omega} \sin(\omega t + \frac{\pi}{2})$<br>For $t = 0$ : $U\sqrt{2} = \frac{I\sqrt{2}}{C\omega} \sin(\frac{\pi}{2}) \Rightarrow C = 3.2 \times 10^{-6} \text{ F} = 3.2 \text{ } \mu\text{F}$ | 3/4   |
| B.1           | Connections of the oscilloscope<br>  | 1/4   |
| B.2           | $u_{AM} = u_{AB} + u_{BD} + u_{DM} \Rightarrow$<br>$U\sqrt{2} \cos \omega t = -LI\omega\sqrt{2} \sin(\omega t + \varphi) + \frac{I\sqrt{2}}{C\omega} \sin(\omega t + \varphi) + RI\sqrt{2} \cos(\omega t + \varphi)$<br>For $\omega t = \frac{\pi}{2} \Rightarrow 0 = -LI\sqrt{2} \cos \varphi + \frac{I\sqrt{2}}{C\omega} \cos \varphi - RI\sqrt{2} \sin \varphi \Rightarrow \tan \varphi = \frac{\frac{1}{C\omega} - L\omega}{R}$   | 1     |
| B.3.a         | $T = 4 \text{ ms} \Rightarrow f = \frac{1}{T} = 250 \text{ Hz}$ .   | 1/2   |
| B.3.b         | $ \varphi  = \frac{2\pi}{8} = \frac{\pi}{4} \text{ rad}$ .  | 1/2   |
| B.4.a         | Current resonance   | 1/4   |
| B.4.b         | $\varphi = 0 \Rightarrow \tan \varphi = 0 \Rightarrow \frac{1}{C\omega_0} = L\omega_0 \Rightarrow LC = \frac{1}{\omega_0^2}$ .  | 1/2   |
| B.5           | $\varphi = \frac{\pi}{4} \Rightarrow 1 = \frac{\frac{1}{C\omega} - L\omega}{R} \Rightarrow C = \frac{1 - LC\omega^2}{R\omega} = 3.2 \times 10^{-6} \text{ F} = 3.2 \text{ } \mu\text{F}$<br>$\Rightarrow LC = \frac{1}{\omega_0^2} \Rightarrow L = \frac{1}{C\omega_0^2} = 32 \text{ mH}$   | 1 1/2 |

**Third exercise (7.5 points)**

| Part of the Q | Answer   | Mark  |
|---------------|--|-------|
| <b>A.I.1</b>  | $\lambda, \lambda_0$ and $n^2$ are positive $\Rightarrow n^2 - 4 > 0 \Rightarrow n > 2 \Rightarrow$ the smallest value is $n = 3$ .  | 1/2   |
| <b>A.I.2</b>  | $\lambda = \lambda_0 \frac{n^2}{n^2 - 4} \Rightarrow \lambda = 656.46 \text{ nm}$ .  | 1/2   |
| <b>A.I.3</b>  | In these conditions:<br>$n = 4$ gives $\lambda = 486.13 \text{ nm}$<br>$n = 5$ gives $\lambda = 435.05 \text{ nm}$<br>$n = 6$ gives $\lambda = 410.17 \text{ nm}$  | 3/4   |
| <b>A.II.1</b> | Ground state $n = 1$ : $E_1 = -13.6 \text{ eV}$ .  | 1/2   |
| <b>A.II.2</b> | 1 <sup>st</sup> energy level (excited): $n = 2$ : $E_2 = -\frac{13.6}{2^2} = -3.4 \text{ eV}$<br>$E_3 = -1.51 \text{ eV}$ ; $E_4 = -0.85 \text{ eV}$ ; $E_5 = -0.54 \text{ eV}$ and $E_6 = -0.38 \text{ eV}$               | 1 1/4 |
| <b>A.II.3</b> | The atom is ionized when $n \rightarrow \infty \Rightarrow E_\infty = 0$   | 1/2   |
| <b>B.1</b>    | $W_0 = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{hc}{W_0} = 5.65 \times 10^{-7} \text{ m} = 565 \text{ nm}$   | 3/4   |
| <b>B.2</b>    | The radiations of Balmer series that can produce photoelectric emission verifies the relation $\lambda < \lambda_0$ ;<br>$H_\beta, H_\gamma$ and $H_\delta$ produce this emission because $\lambda < \lambda_0$            | 1/2   |
| <b>B.3.a</b>  | $P_0 = \frac{P_s \times s}{4\pi D^2} = 2.04 \times 10^{-5} \text{ W}$  | 3/4   |
| <b>B.3.b</b>  | $N_{\%s} = \frac{P_0}{E_{\text{photon}}} = \frac{P_0 \times \lambda_\beta}{h \times c} = 4.99 \times 10^{13} \text{ photons / s}$  | 3/4   |
| <b>B.3.b</b>  | The number of effective photons = number of emitted electrons $N_e$<br>$\Rightarrow N_e = r \times N_0 = 4.37 \times 10^{11} \text{ electrons/s}$<br>$I_0 = \frac{q}{t} = \frac{N_e e}{t} = 6.99 \times 10^{-8} \text{ A}$ | 3/4   |

**Fourth exercise (7.5 points)**

| Part of the Q | Answer   | Mark  |
|---------------|--|-------|
| 1.a           | $(m + m_1) \overrightarrow{OG} = m \overrightarrow{OO} + m_1 \overrightarrow{OM} \Rightarrow \overrightarrow{OG} = \frac{x}{4}$  | 1/2   |
| 1.b           | $I_{(sys)} = I_{(rod)} + I_{(S)} \Rightarrow I = \frac{1}{12} m \ell^2 + \frac{m}{3} x^2 = \frac{m}{12} (4x^2 + \ell^2)$   | 1/2   |
| 2             | $ME = \frac{1}{2} I \theta'^2 - (m + m_1) g OG \cos \theta = \frac{m}{24} (4x^2 + \ell^2) \theta'^2 - \frac{m}{3} g x \cos \theta$   | 3/4   |
| 3.a           | $ME = Cte \Rightarrow \frac{dME}{dt} = 0 \Rightarrow \frac{m}{12} (4x^2 + \ell^2) \theta' \theta'' + \frac{m}{3} g x \theta' \sin \theta = 0, \theta' \neq 0$<br>For small angle $\sin \theta \approx \theta$ (rd) $\Rightarrow \theta'' + \left( \frac{4gx}{4x^2 + \ell^2} \right) \theta = 0$ .  | 3/4   |
| 3.b           | This differential equation has the form:<br>$\theta'' + \omega_0^2 \theta = 0 \Rightarrow \omega_0 = \sqrt{\frac{4gx}{4x^2 + \ell^2}} \Rightarrow T_0 = \sqrt{\frac{4x^2 + \ell^2}{x}}$ .  | 1/2   |
| 4.a           | $\frac{dT_0}{dx} = \frac{1}{2} \left( \frac{4x^2 - \ell^2}{x^2} \right) \left( \frac{4x^2 + \ell^2}{x} \right)^{-\frac{1}{2}}$ ; $T_0$ is minimum when $\frac{dT_0}{dx} = 0$ for $x \in \left] 0, \frac{\ell}{2} \right]$ $\Rightarrow 4x^2 - \ell^2 = 0$ ; then $T_0$ is minimal for $4x^2 = \ell^2 \Rightarrow x = \frac{\ell}{2}$ .   | 1 1/2 |
| 4.b           | $T_0 = \sqrt{\frac{\ell^2 + \ell^2}{\frac{\ell}{2}}} = 2\sqrt{\ell} = 1.41 \text{ s}$  | 1/2   |
| 5.a           | $T_{01} = 2\pi \sqrt{\frac{\ell_1}{g}} = 1.61 \text{ s}$   | 1/2   |
| 5.b.i         | The phenomenon of amplitude resonance will take place when the proper period of the exciter becomes equal (very close) of that of the resonator. As $T_0 = 1.41 \text{ s}$ of (P) is smaller than $T_{01} = 1.61 \text{ S}$ of (P <sub>1</sub> ), therefore the phenomenon of resonance does not take place  | 1/2   |
| 5.b.ii        | (P <sub>1</sub> ) oscillates with large amplitude, therefore it is in resonance of amplitude with (P); and then the proper period of (P) is equal to $T_{01} = 1.61 \text{ s}$ . $4x^2 - (1,61)^2 x + \ell^2 = 0$<br>$\Rightarrow$ The solution of this quadratic equation gives;<br>$x_1 = 53 \text{ cm}$ (rejected because it is $>$ than $\frac{\ell}{2} = 25 \text{ cm}$ ) and $x_2 = 11.75 \text{ cm}$ (accepted) | 1 1/2 |