مسابقة في مادة الفيزياء — الاسم:
المدة ثلاث ساعات — الرقم:

This exam is formed of four exercises in four pages numbered from 1 to 4.
The use of non-programmable calculator is recommended.

First exercise: (7.5 points)

Horizontal mechanical oscillator

A horizontal mechanical oscillator is formed of a puck (S), of mass m = 510 g, attached to two identical springs of un-jointed loops whose other extremities A and B are connected to two fixed supports. Each spring is of negligible mass, natural length ℓ_0 and stiffness $k = 10\ \text{Nm}^{-1}$. (S) may slide along a horizontal air table and its center of inertia G can then move along a horizontal axis x'Ox.

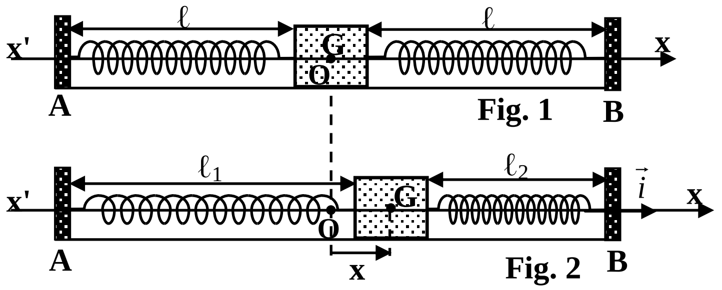


Fig. 1
Fig. 2

At equilibrium (Fig. 1):

- G coincides with the origin O of the axis x'x;
- each spring is elongated by $\Delta\ell$ such that its length is $\ell = \ell_0 + \Delta\ell$.

The horizontal plane passing through G is taken as a reference level of gravitational potential energy.

A – Theoretical Study

(S) is supposed to oscillate without friction. At an instant t, the abscissa of G is $x = \overline{OG}$, the algebraic value of its velocity is $v = \frac{dx}{dt}$ and the two springs have lengths ℓ_1 and ℓ_2 (Fig. 2).

1) **a)** Referring to figure 2, express ℓ_1 and ℓ_2 in terms of ℓ and x.
b) Show that, at an instant t, the total elastic potential energy stored in the two springs is given by: $PE_e = k[(\Delta\ell)^2 + x^2]$.

2) Write down, at an instant t, the expression of the mechanical energy of the system (puck, two springs, Earth) in terms of v, m, k, $\Delta\ell$ and x.

3) Derive the second order differential equation in x that describes the motion of G.

4) The solution of the differential equation is of the form: $x = X_m \cos(\omega_0 t + \varphi)$ where X_m, ω_0 and φ are constants.
a) Determine, in terms of k and m, the expression of ω_0.
b) Deduce the value T_0 of the proper period of the oscillations of G.

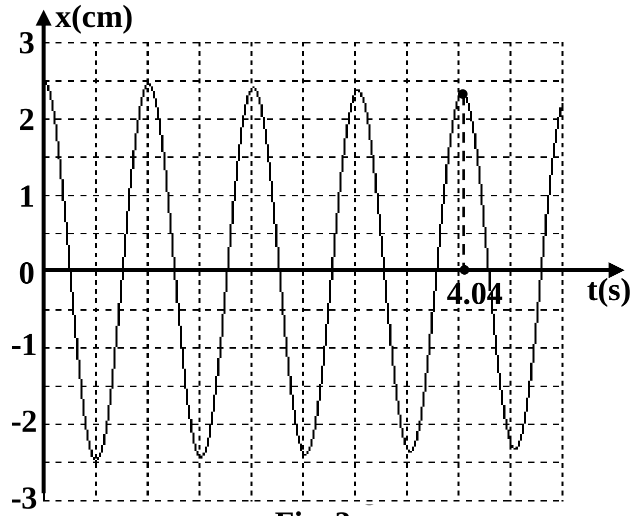


Fig. 3

B – Experimental study

An appropriate apparatus allows the recording of the abscissa x of G as a function of time (Fig. 3)

1) **a)** The experimental value of the period T is slightly different from the theoretical value T_0. Indicate the cause of this difference.
b) Determine, referring to figure 3, the period T of the oscillations of G.

2) At t = 4.04 s, the amplitude of the oscillations is 2.36 cm.
a) Determine the mechanical energy lost by the system (puck, two springs, Earth) between the instants $t_0 = 0$ and t = 4.04 s.
b) Deduce the average power lost in this interval.

3) The extremity A of the left spring is coupled to an exciter (E) of adjustable frequency « f » (Fig. 4). With an appreciable amount of friction, the puck is forced to oscillate on the air table with a frequency equal to that of (E). The variations, as a function of time, of the abscissa x of G is represented for two values of « f » by figures 5 and 6.

a) Determine, in each case, the amplitude and the period of the oscillations of G.

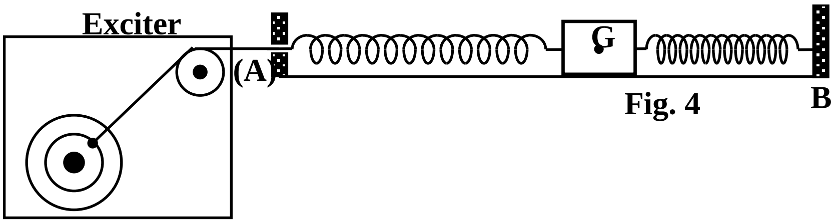


Fig. 4

b) The amplitude of the oscillations represented in figure 6 is larger than that of the oscillations of figure 5. Interpret this increase.

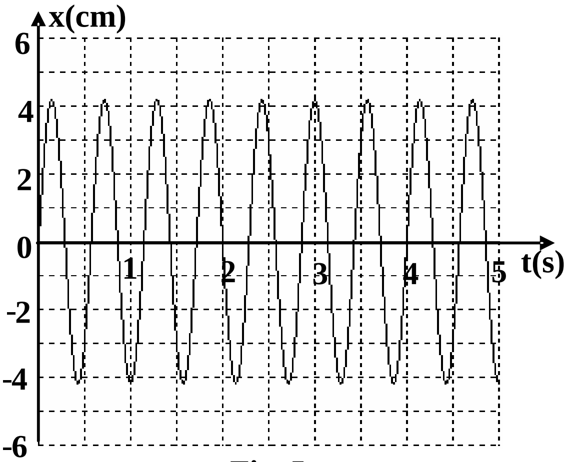


Fig. 5

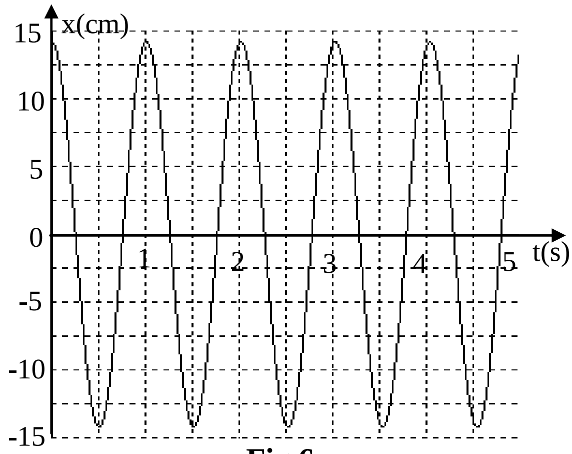


Fig.6

<u>Second exercise</u>: (7.5 points)   Determination of the characteristics of a coil

In order to determine the characteristics of a coil, we consider the electric circuit represented in figure1. This circuit is formed of a capacitor of capacitance C, the coil of inductance L and of resistance r, a resistor of resistance R and an ammeter (A) of negligible resistance, all connected in series across an LFG, of adjustable frequency f, that maintains across its terminals an alternating sinusoidal voltage :

$u = u_{AM} = U\sqrt{2}\,\sin(2\pi ft + \varphi)$.

Thus the circuit carries an alternating sinusoidal current: $i = I\sqrt{2}\,\sin(2\pi ft)$ (Fig.1).

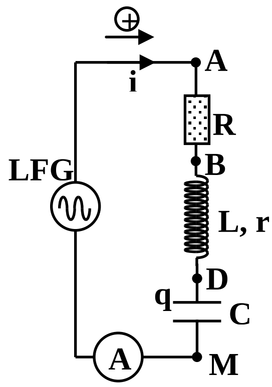


Fig. 1

A – 1) Write the expression of the voltage:

a) u_{AB} across the terminals of the resistor in terms of R, I, f and t;

b) u_{BD} across the terminals of the coil in terms of r, L, I, f and t.

2) Show that the voltage across the terminals of the capacitor is:

$$u_{DM} = -\frac{I\sqrt{2}}{2\pi f\ C}\cos(2\pi ft).$$

B – 1) Applying the law of addition of voltages and giving t two particular values, show that:

a) The effective value of the current is: $I = \dfrac{U}{\sqrt{(R+r)^2 + (2\pi f L - \dfrac{1}{2\pi f C})^2}}$;

b) The phase difference φ between the voltage u_{AM} and the current i is: $\tan\varphi = \dfrac{2\pi f L - \dfrac{1}{2\pi f C}}{R+r}$.

2) U is maintained constant and f is varied; the ammeter indicates a value I for each value of f. An appropriate device allows to plot the curve representing the variation of I as a function of f (Fig. 2).
This curve shows an evidence of a physical phenomenon for $f = f_0 = 110$ Hz.

a) Name this phenomenon.
b) Indicate the value I_0 of I corresponding to the value f_0 of f.
c) For $f = f_0$, show that:
 i) $4\pi^2 f_0^2 LC = 1$ using the relation given in part (B-1-b);
 ii) the circuit is equivalent to a resistor of resistance $R_t = R + r$ using the relation given in part (B-1-a).
d) Calculate the value of L knowing that $C = 21\mu F$.
e) Calculate the resistance r knowing that U = 8 V and R = 30 Ω .

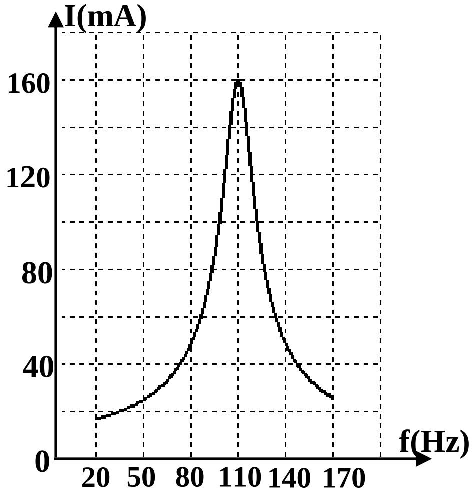


Fig. 2

<u>Third exercise</u>: (7.5 points) **Nuclear submarine**

A nuclear submarine is powered with a nuclear reactor using uranium 235. We intend to determine the efficiency of the reactor of this submarine that consumes 112 g of uranium 235 per day.
Take: $c = 3\times10^{8}$ m/s; 1 MeV = 1.6×10^{-13} J; mass of 1 atom of uranium 235 = 3.9×10^{-25} kg.

1) One of the nuclear reactions that take place inside the reactor is:

$${}_{0}^{1}n + {}_{92}^{235}U \longrightarrow {}_{36}^{91}Kr + {}_{X}^{Y}Ba + 3{}_{0}^{1}n$$

a) Determine the values of X and Y.
b) Is the previous reaction provoked or spontaneous? Justify.
c) Give the condition satisfied by the projectile neutron for this reaction to take place.

2) The adjacent table gives the binding energy per nucleon $\frac{E_B}{A}$ of the involved nuclei.

Nucleus	${}_{92}^{235}U$	${}_{36}^{91}Kr$	${}_{Z}^{A}Ba$
$\frac{E_B}{A}$ (MeV/nucleon)	7.59	8.55	8.31

a) Calculate the binding energy E_B of each nucleus.
b) Write the expression of the binding energy E_B of a nucleus ${}_{Z}^{A}X$ in terms of A, Z, m_X [mass of the nucleus], m_P [mass of a proton] and m_n [mass of a neutron].
c) Show that the energy liberated by this fission reaction is: $E_{lib} = E_B(Kr) + E_B(Ba) - E_B(U)$.
d) Deduce the value of E_{lib} in MeV and in joule.

3) We suppose that the other nuclear fission reactions, that might take place in the reactor, liberate approximately the same amount of energy as that obtained in part (2-d).
a) Calculate the energy liberated by the fission of 112 g of uranium 235.
b) Determine the efficiency of the reactor of the submarine knowing that it delivers an electric power of 25 MW.

<u>Fourth exercise</u>: (7.5 points) **Photoelectric effect**

Given: $c = 3\times10^{8}$ m/s ; 1 eV = 1.6×10^{-19} J ; $h = 6.64\times10^{-34}$ Js.

A – Emission of photoelectrons

Let W_0 be the minimum energy needed to extract an electron from the surface of a metal that covers the cathode of a photo cell and ν_0 the threshold frequency of this metal.

1) Define the threshold frequency ν_0.
2) Write the relation between W_0 and ν_0.
3) To determine W_0 and then the nature of the metal, we illuminate the cathode of the photo cell successively and separately each time with radiations of different frequencies and we determine the maximum kinetic energy (KE_{max}) of the emitted photoelectrons for each radiation of frequency ν. We obtain the results shown in table 1.

ν ($\times10^{14}$ Hz)	5.5	6.2	6.9	7.5
KE_{max}(eV)	0.20	0.49	0.79	1.03

Table (1)

a) Trace the curve representing the variations of KE_{max} in terms of ν.
Scale: on the horizontal axis: 1 cm $\rightarrow 10^{14}$ Hz ; on the vertical axis: 1 cm $\rightarrow$ 0.20 eV.

b) i) The obtained graph confirms with Einstein's relation concerning the photoelectric effect. Justify.
ii) Name the physical constant that is represented by the slope of this graph.

c) Using the graph, determine the value of:
i) this physical constant;
ii) the threshold frequency ν_0.

d) Deduce the value of W_0.

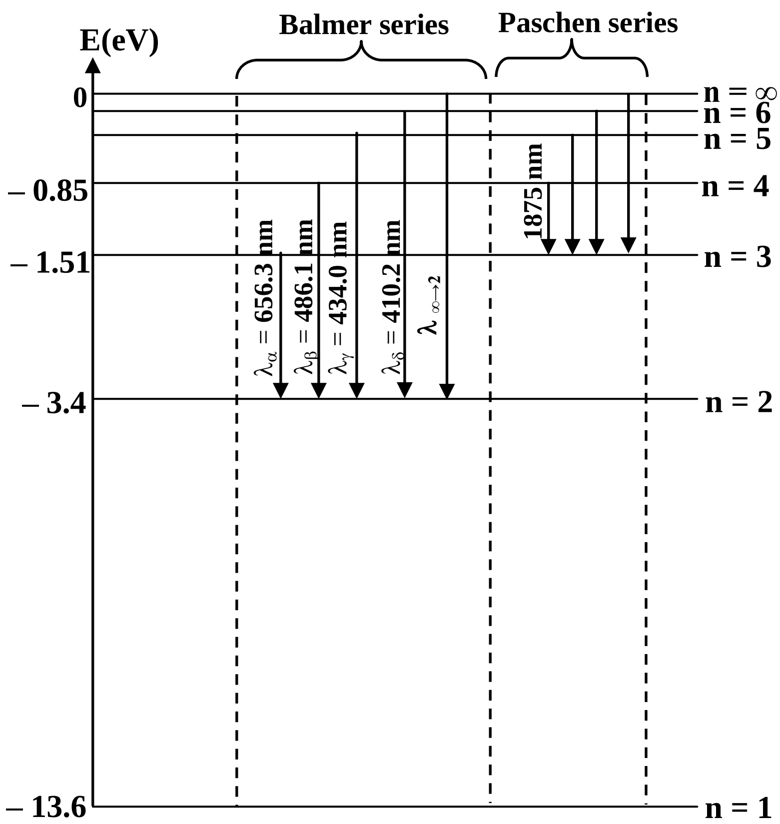
e) Referring to table 2, indicate the nature of the metal used.

metal	cesium	sodium	potassium
W_0 (eV)	2.07	2.28	2.30

Table (2)

B – The hydrogen atom

The spectral lines that constitute the hydrogen spectrum can be classified into many series; each series corresponds to the electronic transitions that lead to the same energy level. The figure below shows two of these series with the wavelengths of some emitted radiations.
The radiations emitted by the hydrogen gas lamp illuminates the cesium cathode of a photo cell.



1) Consider the spectral line having the smallest wavelength of the Paschen series.
a) To which transition does this line correspond?
b) Deduce the energy of the corresponding emitted photon.
c) Can the photons of the Paschen series extract photoelectrons from a cesium surface? Why?

2) Consider the spectral lines corresponding to the emitted radiations of wavelengths λ_α and λ_β of Balmer series.
a) Referring to the above energy diagram, calculate the corresponding frequencies ν_α and ν_β .
b) One of these two radiations can extract photoelectrons from the surface of cesium. Specify this radiation.

First exercise: (7.5 points)

Part of the Q	Answer	Mark
A.1.a	$\ell_1 = \ell + x$ and $\ell_2 = \ell - x$	½
A.1.b	The elastic potential energy stored in the two springs is: $PE_e = \frac{1}{2} k(\Delta\ell + x)^2 + \frac{1}{2} k(\Delta\ell - x)^2 = k(\Delta\ell^2 + x^2)$	1
A.2	The mechanical energy of the system is: $ME = \frac{1}{2} m v^2 + k(\Delta\ell^2 + x^2)$	½
A.3	No Friction $\Rightarrow \frac{dME}{dt} = 0 \Rightarrow mvv' + 2kxx' = 0 \Rightarrow mx'' + 2kx = 0$ $\Rightarrow x'' + \frac{2k}{m} x = 0.$	1
A.4.a	$x' = -\omega_0 X_m \sin(\omega_0 t + \varphi)$ and $x'' = -\omega_0^2 X_m \cos(\omega_0 t + \varphi)$. By replacing in the differential equation: $\omega_0^2 = \frac{2k}{m}$. $\omega_0 = \sqrt{\frac{2k}{m}}$	½
A.4.b	The value of the proper period T_0 is: $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{2k}} = 2\pi\sqrt{\frac{0.51}{2\times10}} = 1.003\ s \approx 1\ s$	½
B.1.a	T is slightly greater than T_0 because of friction.	½
B.1.b	$4T = 4.04 \Rightarrow T = 1.01\ s$	¼
B.2.a	Variation of the mechanical energy: $ME_0 = k(\Delta\ell^2 + x_0^2)$ at $t = 0s$ $ME_{4T} = k(\Delta\ell^2 + x_4^2)$ at $t = 4T$ $\Rightarrow \Delta ME = k[x_4^2 - x_0^2] = 10[5.57\times10^{-4} - 6.25\times10^{-4}] = -6.8\times10^{-4}$ J. So the loss is 6.8×10^{-4} J.	¾
B.2.b	The lost average power: $\frac{\lvert\Delta ME\rvert}{\Delta t} = \frac{6.8\times10^{-4}}{4.04} = 1.68\times10^{-4}$ W	½
B.3.a	Figure 5 : $X_m = 4.1$ cm and $T = \frac{4}{7} = 0.57$ s. Figure 6 : $X_m = 14$ cm and $T = 1.01$ s.	1
B.3.b	In the case (figure 5) we are far from the resonance $T < T_0$ and in the case (figure 6) there is a resonance $T = T_0$.	½

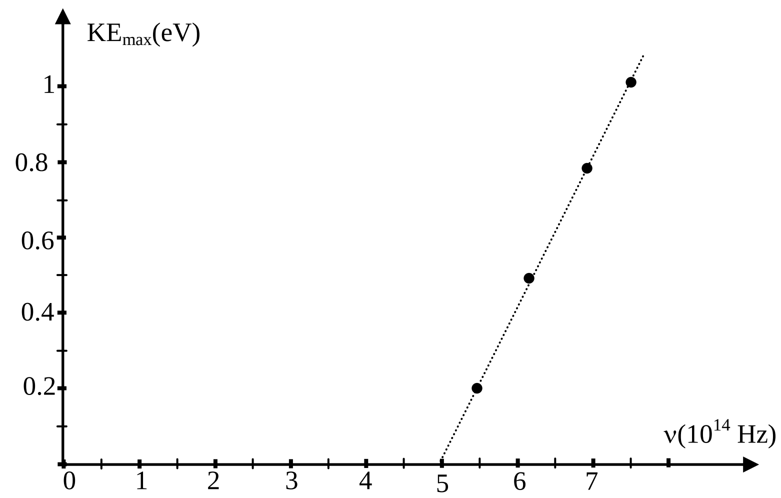
Second exercise: (7.5 points)

Part of the Q	Answer	Mark
A.1.a	$u_{AB} = Ri = RI\sqrt{2}\ \sin(2\pi ft)$	½
A.1.b	$u_{BD} = L\frac{di}{dt} + ri = 2\pi fL\ I\sqrt{2}\cos(2\pi ft) + rI\sqrt{2}\ \sin(2\pi ft)$	½
A.2	$u_{DM} = \frac{q}{C}$ but $dq = idt = I\sqrt{2}\ \sin(2\pi ft)dt \Rightarrow q = -\frac{I\sqrt{2}}{2\pi f}\cos(2\pi ft)$. $\Rightarrow u_{DM} = -\frac{I\sqrt{2}}{2\pi f\ C}\cos(2\pi ft)$.	1
B.1.a	$u_{AB} = u_{AB} + u_{BD} + u_{DM}$ $U\sqrt{2}\ \sin(2\pi ft + \varphi) = (2\pi fL - \frac{1}{2\pi f C})I\sqrt{2}\cos(2\pi ft) + (R + r)\ I\sqrt{2}\ \sin(2\pi ft)$ For $2\pi ft = 0$ we get: $U\sin\varphi = I\,(2\pi f_0\ L - \frac{1}{2\pi f C})$ .... (1) For $2\pi ft = \frac{\pi}{2}$ we get: $U\cos\varphi = I(R+r)$ …….. (2) Squaring and adding (1) and (2), we get: $U^2 = \left[(R+r)^2 + (2\pi f L - \frac{1}{2\pi f C})^2\right]I^2$ $\Rightarrow I = \frac{U}{\sqrt{(R+r)^2 + (2\pi f L - \frac{1}{2\pi f C})^2}}$.	2
B.1.b	The ratio $\frac{(1)}{(2)}$) gives: $\tan\varphi = \frac{2\pi f L - \frac{1}{2\pi f C}}{R+r}$.	½
B.2.a	Current resonance	¼
B.2.b	$I = I_0 = 160$ mA.	½
B.2.c.i	According to the relation (B-1-b) , i and u_{AM} are in phase $\Rightarrow \varphi = 0 \Rightarrow \tan\varphi = 0 \Rightarrow 2\pi f_0\ L - \frac{1}{2\pi f_0 C} = 0 \Rightarrow 4\pi^2 f_0^2\ LC = 1$	¾
B.2.c.ii	The relation (B-1-a) becomes $U = (R + r)I \Rightarrow$ the circuit is equivalent to a resistor of resistance $R_t = R + r$.	½
B.2.d	The relation $4\pi^2 f_0^2\ LC = 1 \Rightarrow L = 0.1$ H	½
B.2.e	At resonance the expression $U = (R + r)I \Rightarrow (R + r) = 50 \Rightarrow r = 20\ \Omega$	½

Third exercise: (7.5 points)

Part of the Q	Answer	Mark
1.a	Conservation of mass number: 1 + 235 = 91 + Y + 3 $\Rightarrow$ Y = 142 Conservation of charge number: 0 + 92 = 36 + X + 0 $\Rightarrow$ X = 56	**1**
1.b	The nuclear reaction is provoked, because it needs an external intervention : bombarded with a thermal neutron	**½**
1.c	The neutron must be thermal (low kinetic energy)	**½**
2.a	Since E_B (X) = A $\frac{E_{B(X)}}{A}$, then: E_B (U)= 235×7.59 = 1783.65 MeV; E_B (Kr) = 91×8.55 = 778.05 MeV; E_B (Ba) = 142×8.31 = 1180.02 MeV.	**1**
2.b	$E_{B(X)} = [Z\times m_P + (A - Z)m_n - m_X]c^2$	**½**
2.c	$m_X = [Z\times m_P + (A - Z)m_n] - \frac{E_{B(X)}}{c^2}$ $E_{lib} = \Big\{ [m_n + (92\,m_P + (235 - 92)m_n - \frac{E_{B(U)}}{c^2}\,]$ $- [(36\,m_P + (91 - 36)m_n - \frac{E_{B(Kr)}}{c^2})\,]$ $- [(56\,m_P + (142 - 56)m_n - \frac{E_{B(Ba)}}{c^2})]$ $- (3m_n) \Big\} c^2$ $E_{lib} = E_B\,(Kr) + E_B(Ba) - E_B\,(U)$	**1½**
2.d	E_{lib} = 1180.02 + 778.05 – 1783.65 = 174.42 MeV. $E_{lib} = 174.42\times1.6\times10^{-13} = 2.79\times10^{-11}$ J	**¾**
3.a	1 fission reaction $\rightarrow 3.9\times10^{-25}$ kg $\rightarrow 2.79\times10^{-11}$ J 0.112 kg $\rightarrow$? The energy liberated by the fission of 112 g is: 8.0123×10^{12} J.	**¾**
3.b	$P = \frac{E}{t} = \frac{8.0123\times10^{12}}{24\times3600} = 9.2735\times10^{7}$ watt The efficiency of the reactor is : $\xi = \frac{25\times10^{6}}{92.735\times10^{6}} = 0.269 = 26.9\%$	**1**

Fourth exercise (7.5 points)

Part of the Q	Answer	Mark
A.1	The threshold frequency ν_0 of a metal is the minimum frequency of an electromagnetic wave that can extract an electron when it illuminates the metal.	**½**
A.2	$W_0 = h\ \nu_0$.	**¼**
A.3.a		**1½**
A.3.b.i	$h\nu = W_0 + KE_{max} \Rightarrow KE_{max} = h\nu - h\nu_0$ which is a linear function of the frequency ν.	**½**
A.3.b.ii	The slope of the graph is h (Planck's constant)	**¼**
A.3.c.i	$h = \dfrac{\Delta KE_{max}}{\Delta\nu} \Rightarrow h = \dfrac{(1.03-0.2)\times1.6\times10^{-19}}{(7.5-5.5)\times10^{14}} = 6.64\times10^{-34}$ Js	**1**
A.3.c.ii	$\nu_0 = 5\times10^{14}$ Hz,	**¼**
A.3.d	$W_0 = h\nu_0 = 6.64\times10^{-34}\times5\times10^{14} = 3.32\times10^{-19}$ J ; $W_0 = \dfrac{3.32\times10^{-19}}{1.6\times10^{-19}} = 2.075$ eV	**½**
A.3.e	The used metal is cesium.	**¼**
B.1.a	The smallest wavelength in the Paschen series corresponds to the transition from $n = \infty$ to $n = 3$.	**½**
B.1.b	The energy of the corresponding photon is: $E_\infty - E_3 = 1.51$ eV.	**½**
B.1.c	No, because the maximum of energy of the emitted photon in Paschen series is 1.51 eV which is less than 2.075 eV	**½**
B.2.a	We know that $\upsilon = \dfrac{c}{\lambda}$ $\Rightarrow \nu_\alpha = \dfrac{3\times10^8}{656.3\times10^{-9}} = 4.57\times10^{14}$ Hz.; $\nu_\beta = \dfrac{3\times10^8}{486.10\times10^{-9}} = 6.17\times10^{14}$ Hz	**½**
B.2.b	The radiation of frequency $\nu_\alpha < \nu_0$ does not emit photoelectrons; while$\nu_\beta > \nu_0$ emits photoelectrons.	**½**