This exam is formed of four exercises in four pages numbered from 1 to 4
The use of non-programmable calculator is recommended

First exercise: (7 ½ points)

Verification of Newton's Second Law

Consider an inclined plane that makes an angle $\alpha = 30^o$ with the horizontal plane.
An object (S), supposed as a particle, of mass $m = 0.5$ kg is launched from the bottom O of the inclined plane, at the instant $t_0 = 0$, with a velocity $\vec{V}_0 = V_0 \vec{i}$ along the line of the greatest slope (OB).
Let A be a point of OB such that OA = 5 m (fig.1).
The position of (S), at the instant t, is given by $\overrightarrow{OM} = x\vec{i}$ where $x = f(t)$.

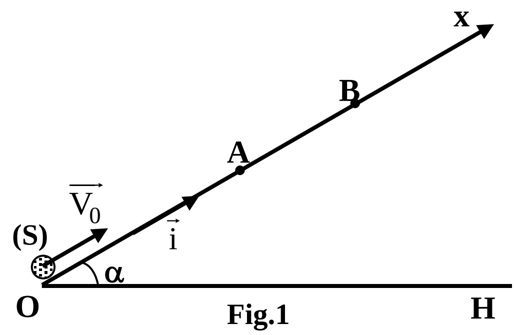

Fig.1

The variation of the mechanical energy of the system [(S), Earth], as a function of x, is represented by the graph of figure 2.
Take:
- The horizontal plane passing through OH as a gravitational potential energy reference;
- $g = 10\ ms^{-2}$.

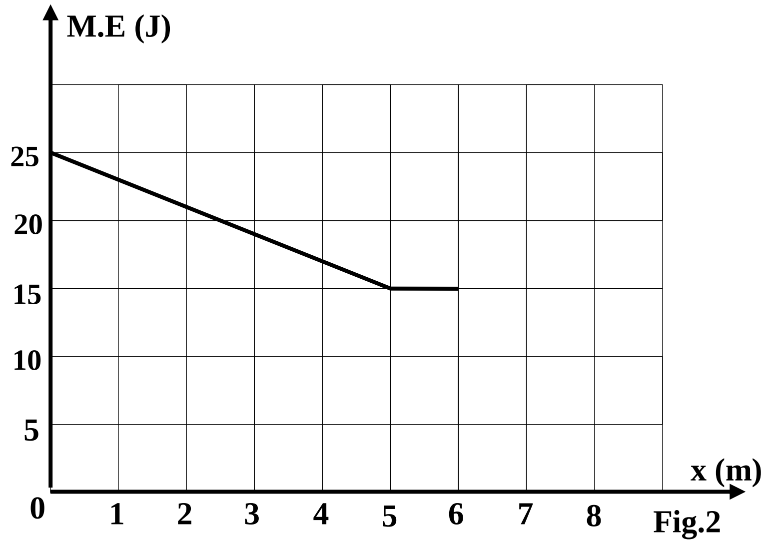

Fig.2

1) Using the graph of figure 2:
   a) show that (S) is submitted to a force of friction between the points of abscissas $x_0 = 0$ and $x_A = 5$ m;
   b) **i)** calculate the variation of the mechanical energy of the system [(S), Earth] between the instants of the passage of (S) through the points O and A;
   ii) deduce the magnitude of the force of friction, supposed constant, between O and A;
   c) determine, for $0 \le x \le 5$ m**,** the expression of the mechanical energy of the system [(S),Earth] as function of x;
   d) Determine the speed of (S) at the point of abscissa x = 6 m.

2) Let v be the speed of (S) when it passes through the point M of abscissa x so that $0 \le x \le 5$ m**.**
   a) Determine the relation between v and x.
   b) Deduce that the algebraic value of the acceleration of (S) is $a = -9\ ms^{-2}$.

3) **a)** Determine the values of the speed of (S) at O and at A.
   b) Calculate the duration $\Delta t = t_A - t_0 = t$ of the displacement of (S) from O to A, knowing that the algebraic value of the velocity of (S) is given by: $v = at + v_0$.
   c) Determine the linear momentums $\vec{P}_O$ and $\vec{P}_A$ of (S), at O and at A respectively.

4) Determine the resultant of the external forces $\sum \overrightarrow{F_{ext}}$ acting on (S).

5) Verify, using the previous results, Newton's second law knowing that $\frac{\Delta \vec{P}}{\Delta t} = \frac{d\vec{P}}{dt}$.

Second exercise: (7 ½ points)

Torsion Pendulum

The aim of this exercise is to study the motion of a torsion pendulum in three different situations.

Consider a torsion pendulum that is constituted of a homogenous disk (D), of negligible thickness, suspended from its center of gravity O by a vertical torsion wire connected at its upper extremity to a fixed point O' (fig. 1).

Given :

- torsion constant of the wire : C = 0.16 m.N/rad ;
- moment of inertia of the disk with respect to the axis OO': $I = 25\times10^{-4}$ kg·m^2.

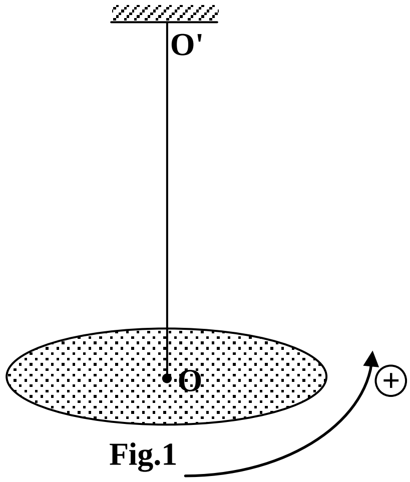

Fig.1

A – Free un-damped oscillations

The disk is in its equilibrium position. It is rotated around OO', in a direction considered positive, by an angle $\theta_m = 0.1$ rad (fig.1). The disk is then released without initial velocity at the instant $t_0 = 0$.

Take the horizontal plane passing through O as a gravitational potential energy reference.

At the instant t, the angular abscissa of the disk is θ and its angular velocity is $\theta' = \frac{d\theta}{dt}$.

1) Write the expression of the mechanical energy M.E of the system (pendulum, Earth) in terms of I, θ, C and θ'.

2) Suppose that the forces of friction are negligible.

- **a)** Derive the differential equation, in θ, that describes the motion of the disk.
- **b)** The time equation of the motion of the disk has the form: $\theta = \theta_m \sin(\omega_0 t + \phi)$. Determine ω_0 and ϕ.
- **c)** Determine the angular velocity of the disk when it passes through its equilibrium position for the first time.

B – Free damped oscillations

In reality, the disk is subjected to a force of friction whose moment with respect to OO': $\mathcal{M} = -h\theta'$ where h is a positive constant.

1) Applying the theorem of angular momentum on the disk, show that the differential equation , in θ, describing its motion is written as: $\theta'' + \frac{h}{I}\theta' + \frac{C}{I}\theta = 0$.

2) Determine, in terms of h and θ', the expression $\frac{dM.E}{dt}$ (the derivative, with respect to time, of the mechanical energy M.E of the system [pendulum, Earth]). Deduce the sense of the variation of M.E.

C – Forced oscillations

The pendulum is at rest and at its equilibrium position. An exciter (E), coupled to the disk, provides it with periodic excitations of adjustable pulsation ω_e.

When we vary ω_e of (E), the amplitude θ_m of motion of the disk takes a maximum value of 0.25 rad for $\omega_e = \omega_r$.

1) Name the physical phenomenon that takes place.

2) Indicate the approximate value of ω_r.

3) Sketch the shape of the curve that represents the variation of the amplitude θ_m as a function of ω_e.

<u>Third exercise</u>: (7 ½ points)

Determination of the characteristics of an unknown component

An electric component (D), of unknown nature, may be a resistor of resistance R', or a coil of inductance L and of resistance r or a capacitor of capacitance C.
To determine its nature and its characteristics, we connect it in series with a resistor of resistance $R = 10\ \Omega$ across a generator G as shown in figure 1.
An oscilloscope is connected so as to display the voltage $u_g = u_{AM}$ across the generator and the voltage $u_R = u_{BM}$ across the resistor.

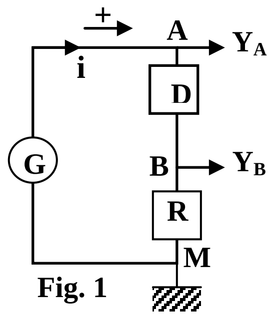

Fig. 1

A – Case of a DC voltage

The generator G delivers a constant voltage U_0. On the screen of the oscilloscope we observe the oscillograms of figure 2.

1) Prove that the voltage $U_0 = 12$ V.

2) **a)** Determine, in the steady state, the value I of the current in the circuit.

b) Deduce that (D) is not a capacitor.

c) Determine the resistance of the component (D).

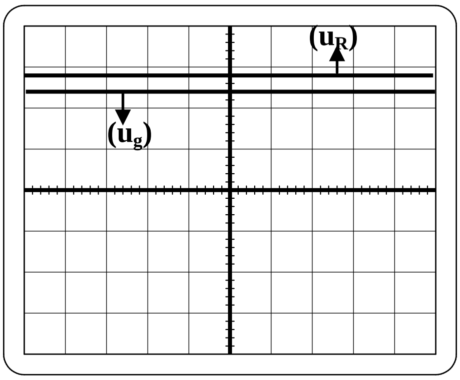

Fig.2 **Channel A: S_V = 5 V/div**
Channel B: S_V = 2 V/div

B – Case of an AC voltage

The generator G delivers now an alternating sinusoidal voltage. On the screen of the oscilloscope we observe the waveforms of figure 3.

1) Referring to the waveforms of figure 3, show that:

a) (D) is a coil;

b) the waveform (2) represents the variation of the voltage u_R across the resistor.

2) The voltage across the generator is given by: $u_g = U_m\sin(\omega t)$. Determine U_m and ω.

3) Determine the expression of i as a function of time.

4) Applying the law of addition of voltages and giving t two particular values, determine the inductance L and the resistance r of (D).

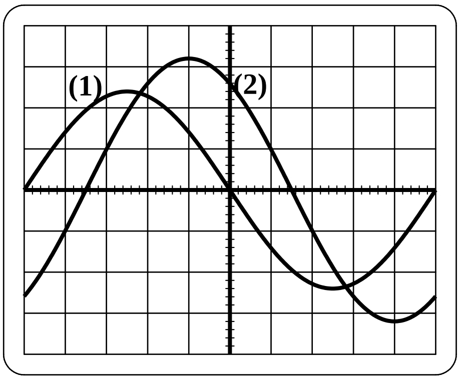

Fig.3 **Channel A: S_V = 5 V/div**
Channel B: S_V = 1 V/div
Time base: S_h = 2 ms/div

5) To verify the values of L and r of (D), we add a capacitor of adjustable capacitance C in series to the previous circuit.
For $C = 10^{-4}$ F, we obtain the waveforms of figure 4.

a) Name the observed phenomenon.

b) Verify, using the waveforms of figure 4, the values of L and r.

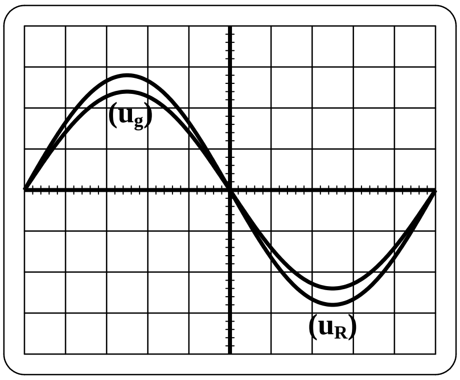

Fig.4 **Channel A: S_V = 5 V/div**
Channel B: S_V = 2 V/div
Time base: S_h = 2 ms/div

<u>Fourth exercise</u>: (7 ½ points)

Nuclear Fission

The nuclear chain fission reaction, conveniently controlled in a nuclear power plant, can be a source of a huge amount of energy able to generate electric power.

Given:

Masses of nuclei: ${}^{235}_{92}U$ = 234.9934 u; ${}^{138}_{x}Ba$ = 137.8742 u; ${}^{y}_{36}Kr$ = 94.8871 u;

molar mass of ${}^{235}U$ = 235 g mol^{-1}; Avogadro's number $N_A = 6.02\times10^{23}$ mol^{-1};

$m({}^{1}_{0}n)$ = 1.0087 u; 1 u = 931.5 MeV/c^2; 1 eV = 1.6×10^{-19} J.

A – Efficiency of a nuclear power plant

In the reactor of a nuclear power plant, we use natural uranium enriched with uranium 235. The nucleus of a uranium 235 captures a thermal neutron and is transformed into a nucleus of uranium 236 in an excited state. The decay of this nucleus is accompanied by the emission of a photon γ of energy equal to 20 MeV.

1) **a)** Complete the following reaction: ${}^{236}_{92}U^{*} \rightarrow \; ..... \; + \; \gamma$

b) Indicate the value of the excess of energy possessed by a uranium 236 nucleus in the excited state.

2) The obtained uranium nucleus, breaks instantaneously, producing two nuclides called fission fragments with the emission of some neutrons and γ photon, so the overall equation is:

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{138}_{x}Ba + {}^{y}_{36}Kr + 3\,{}^{1}_{0}n + \gamma $$

Determine:

a) x and y;

b) in MeV, the energy liberated by the fission of a uranium 235 nucleus;

c) the energy liberated by the fission of 1 g of uranium 235;

d) the efficiency of the nuclear power plant, knowing that it provides an electric power of 800 MW and consumes 2.8 kg of uranium 235 per day.

B – Chain Reaction

The kinetic energy of a neutron that may produce the fission of uranium 235 nucleus should be of the order of 0.04 eV.

We suppose that all the neutrons emitted by the fission reactions have the same kinetic energy.

1) The sum of the kinetic energies of the two fragments (Kr and Ba) is equal to 174 MeV and the energy of the emitted γ photon is E_γ = 20 MeV.

a) Show, using the conservation of the total energy, that the kinetic energy of a neutron emitted by this fission is 2 MeV.

b) Deduce that the emitted neutrons cannot produce fission reactions of uranium 235.

2) To produce a fission by an emitted neutron, it is necessary to slow it down by collisions with carbon 12 atoms in graphite rods. We suppose that each collision between a neutron and one carbon 12 atom is perfectly elastic and that the velocities before and after collision are collinear.

Take: $m({}^{1}_{0}n)$ = 1 u and $m({}^{12}C)$ = 12 u.

a) Let $\overrightarrow{V_0}$ be the velocity of one emitted neutron just before collision and $\overrightarrow{V_1}$ its velocity just after its first collision with a carbon 12 atom supposed initially at rest. Show that: $\left|\frac{V_1}{V_0}\right| = k = \frac{11}{13}$.

b) **i)** Show that the ratio of the kinetic energies just after and just before the first collision of the emitted neutron is: $\frac{K.E_1}{K.E_0} = k^2$.

ii) Determine the number of collisions needed for an emitted neutron with carbon 12 atoms, to slow down so that its kinetic energy is reduced to 0.04 eV.

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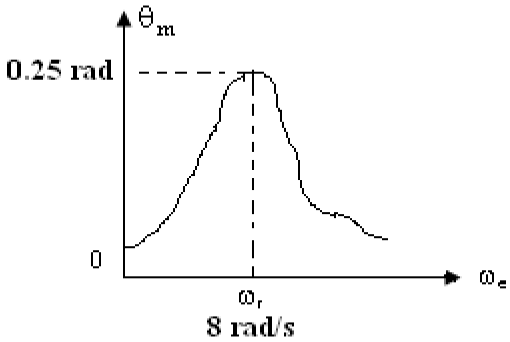
First exercise: (7 ½ points)

Part of the Q	Answer	Mark
1.a	Since the mechanical energy decreases along this part	0.25
1.b.i	$\Delta ME = ME_f - ME_i = 15 - 25 = -10$ J.	0.50
1.b.ii	$\Delta ME = W(\vec{f}) = -f\,x \Rightarrow f = \frac{10}{5} = 2$ N.	0.75
1.c	$ME = ax + b$; For $x = 0 \Rightarrow M.E = 25$ J $\Rightarrow b = 25$ And $a = \frac{\Delta ME}{\Delta x} = \frac{-10}{5} = -2$ J/m $\Rightarrow M.E = -2x + 25$.(ME in J; x in m)	0.75
1.d	$ME = mgh + \frac{1}{2}mV^2 = mgx\sin\alpha + \frac{1}{2}mV^2 = 0.5\times10\times6\times0.5 + \frac{1}{2}mV^2$ then $15 = 15 + \frac{1}{2}mV^2 \Rightarrow V = 0$.	0.75
2.a	$ME = \frac{1}{2}mV^2 + mgx\sin\alpha = -2x + 25 \Rightarrow 0.25\,V^2 + 4.5x - 25 = 0$.	0.5
2.b	Derive with respect to time : $\Rightarrow 0.5Va + 4.5V = 0 \Rightarrow a = -9$ ms^{-2}.	0.50
3.a	Speed at O: $(PE_g)_O = 0$; $M.E = 25 = \frac{1}{2}mV_0^2 \Rightarrow V_0 = 10$ ms^{-1}. Speed at A: $ME = \frac{1}{2}mV_A^2 + mgx_A\sin\alpha = 15$ $\Rightarrow V_A = \sqrt{10} = 3.16$ ms^{-1}.	1.00
3.b	$\vec{P}_0 = m\vec{V}_0 \Rightarrow P_0 = 0.5\times10 = 5$ kg·ms^{-1}. $\vec{P}_A = m\vec{V}_A \Rightarrow P_A = 0.5\times3.16 = 1.58$ kg·ms^{-1}	0.50
3.c	$v = at + v_0 \Rightarrow t = \frac{3.16 - 10}{-9} = 0.76$ s.	0.50
4	$\sum\vec{F}_{ext.} = \vec{N} + m\vec{g} + \vec{f}$ project along the direction of its motion Then $\sum\vec{F}_{ext} = (-mg\sin\alpha - f)\,\vec{i} = -4.5\,\vec{i}$.	0.75
5	$\frac{\Delta\vec{P}}{\Delta t} = \frac{1.58 - 5}{0.76} = -4.5\,\vec{i}$. thus $\frac{\Delta\vec{P}}{\Delta t} = \sum\vec{F}_{ext}$.	0.75

<u>Second exercise</u>: (7 ½ points)

Part	solution	Note
A-1	$ME = PE_e + kE + PE_g = ½\, I\,\theta'^2 + ½\, C\,\theta^2 + 0$	**0.75**
A-2.a	$\frac{dME}{dt} = 0 = I\,\theta'\theta'' + C\,\theta\theta' \Rightarrow \theta'' + \frac{C}{I}\theta = 0$.	**0.75**
A-2.b	$\theta = \theta_m \sin(\omega_0 t + \varphi)$; $\theta' = \omega_0\theta_m \cos(\omega_0 t + \varphi)$; $\theta'' = -\omega_0^2\theta_m \sin(\omega_0 t + \varphi)$ $\theta'' + \frac{C}{I}\theta = 0$ replacing θ" and θ in the differential equation then $\omega_0 = \sqrt{\frac{C}{I}} = 8$ rad/s for $t_0 = 0$, $\theta = \theta m \sin\varphi = \theta_m \Rightarrow \sin\varphi = 1 \Rightarrow \varphi = \frac{\pi}{2}$ rad.	**2**
A-2.c	When the disk passes θ = 0 ⇒ $\sin(\omega_0 t + \varphi) = 0 \Rightarrow \cos(\omega_0 t + \varphi) = \pm 1 \Rightarrow$ $\theta' = \pm\, \omega_0\theta_m$ it passes for the first time in the negative sense $\Rightarrow \theta'_0 = -\,\omega_0\,\theta_m = -\,0.8$ rad/s	**1**
B-1	$\frac{d\sigma}{dt} = \sum M \Rightarrow I\theta'' = -\,C\,\theta\, -\, h\theta' + \overset{0}{\cancel{\vec{M}_{mg} + \vec{M}_T}} \Rightarrow \theta'' + \frac{h}{I}\theta' + \frac{C}{I}\theta = 0$.	**0.75**
B-2	$\frac{dME}{dt} = I\theta'\theta'' + C\theta\theta'$; by replacing θ" we obtain : $\frac{dME}{dt} = I\theta'(-\frac{h}{I}\theta' - \frac{C}{I}\theta) + C\theta\theta' = -\,h\theta'^2$. $\frac{dME}{dt} < 0 \Rightarrow E_m$ decreases with time	**1.25**
C-1	Amplitude resonance	**0.25**
C-2	For $\omega_e = \omega_r = \omega_0 = 8$ rad/s.	**0.25**
C-3		**0.5**

Third exercise: (7 ½ points)

Part of the Q	Answer	Mark
A.1	The voltage U_0 = 5 V/div×2.4 div = 12 V.	0.50
A.2.a	$u_R = RI$, u_R = 2 V/div×2.8 div = 5.6 V $\Rightarrow I = \frac{5.6}{10}$ = 0.56 A.	0.50
A.2.b	This result allows us to eliminate the capacitor since there is passage of a current in the circuit in the steady state.($I \neq 0$)	0.50
A.2.c	Under a DC voltage, a coil behaves in the steady state as a resistor ($L\frac{di}{dt} = 0$), thus if (D) is a coil or a resistor, we can determine the resistance x of (D). $U_g = (R+x)I \Rightarrow R + x = 12/0.56 = 21.43\ \Omega$. Thus x = 21.43 – 10 = 11.43 Ω.	0.75
B.1.a	(D) cannot be a resistor, it is a coil because there is a phase difference between u_g and u_R. ($\phi \neq 0$)	0.25
B.1.b	Since (D) is a coil, the current i will be in lag of phase with u_g and then u_R lags u_g. The waveform (2) represents then the variations of the voltage u_R across the resistor.	0.50
B.2	U_m = 5V/div×2.4 div = 12 V, and $\omega = \frac{2\pi}{T}$; $T = S_h \times x$ = 2ms/div×10 div = 20 ms $\Rightarrow \omega = 100\pi$ rd/s or 314.16 rd/s $u_g = 12\sin(100\pi\tau\ t)$.	0.75
B.3	$I_m = \frac{U_m(R)}{R} = \frac{1\times3.2}{10} = 0.32$ A and $\varphi = \frac{2\pi\times1.5}{10}$; $\varphi = 0.3\pi = 0.94$ rd. $i = 0.32\sin(\omega t - 0.94)$.	0.75
B.4	$u_{coil} = L\frac{di}{dt} + ri$ $u_{coil} = L\times100\pi\times0.32\cos(\omega t - 0.3\pi) + 0.32r\sin(\omega t - 0.3\pi)$. $u_g = 12\sin(\omega t) = L\times100.5\cos(\omega t - 0.3\pi) + (R+r)0.32\sin(\omega t - 0.3\pi)$. **For $\omega t = 0$** $\Rightarrow 0 = 100.5\,L\cos(0.3\pi) - (R+r)0.32\sin(0.3\pi)$ $\Rightarrow L = 0.0044\,(R+r)$ **For $\omega t = \frac{\pi}{2}$** $\Rightarrow 12 = 100.5L\sin(0.2\pi)+0.32(R+r)\cos(0.2\pi)$ $12 = 81.30L + 0.188(R+r) = (0.358 + 0.188)(R+r) = 0.546\,(R+r)$ $\Rightarrow R+r = 21.97\ \Omega \Rightarrow r = 11.97\ \Omega$. $L = 0.0044\times21.97 = 0.097$ H.	2.00
B.5.a	We observe the phenomenon of resonance.	0.25
B.5.b	At the resonance, $T = T_0 = 2\pi\sqrt{LC}$ = 20 ms = 2×10^{-2} s $\Rightarrow 10^{-4} = \pi^2 LC = \pi^2 L\times10^{-4}$ $\Rightarrow L = 0.1$ H At the resonance: $U_m(G) = 12\ V = (R+r)I_m$, $U_m(R)$ = 2V/div×2.8 = 5.6 V = $RI_m \Rightarrow I_m = 0.56$ A So, $R+r = \frac{12}{0.56} = 21.43\ \Omega \Rightarrow r = 11.43\ \Omega$.	0.75

<u>Fourth exercise</u>: (7 ½ points)

Part of the Q	Answer	Mark
A.1.a	${}^{236}_{92}U^* \rightarrow {}^{236}_{92}U + \gamma$	0.25
A.1.b	The excess of energy is 20 MeV	0.25
A.2.a	Conservation of mass number: 1 + 235 = 138 + y + 3 $\Rightarrow$ y = 95. Conservation of charge number: 92 = x + 36 $\Rightarrow$ x = 56	0.75
A.2.b	$\Delta m = 1.0087 + 234.9934 - 137.8742 - 94.8871 - 3\times1.0087 = 0.2147$ u $E = \Delta m\, c^2$ Then $E = 0.2147\times931.5$ MeV/$c^2\times c^2 = 199.99 \approx 200$ MeV	1.00
A.2.c	Number of nuclei contained in 1 g of uranium 235 : $n = \frac{m}{M}N_A = \frac{1}{235}6.02\times10^{23} = 2.56\times10^{21}$ nuclei. The nuclear energy liberated by 1 g: $2.56\times10^{21}\times200\times1.6\times10^{-13} = 8.19\times10^{10}$ J.	1.25
A.2.d	The nuclear energy liberated each day: $2800\times8.19\times10^{10} = 2.29\times10^{14}$ J. The electric energy provided each day: $8\times10^{8}\times24\times3600 = 6.91\times10^{13}$ J Efficiency of the plant: $\frac{6.91\times10^{13}}{2.29\times10^{14}} = 0.30$ or 30%.	0.75
B.1	$m({}^1_0n)\cdot c^2 + KE({}^1_0n) + m({}^{235}U) + KE({}^{235}U)$ $= m(B_a).c^2 + KE(B_a) + m(K_r)\cdot c^2 + KE(K_r) + 3m({}^1_0n)\cdot c^2 + KE({}^1_0n)_{emitted} + E(\gamma)$ $\Rightarrow E_{liberated} = KE(Ba) + KE(K_r) + 3KE({}^1_0n)_{emitted} + E(\gamma) - KE_C({}^1_0n)_{incident}$ $\Rightarrow E_C({}^1_0n)_{emitted} = 1$ MeV.	1.00
B.2.a	Conservation of linear momentum: $m_1\vec{V}_0 = m_1\vec{V}_1 + m_2\vec{V}_2$; collinear $\Rightarrow m_1(V_0 - V_1) = m_2V_2$ (1) The collision is elastic, thus $\frac{1}{2}m_1V_0^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$; $\Rightarrow m_1(V_0^2 - V_1^2) = m_2V_2^2$ (2) $\frac{(2)}{(1)} \Rightarrow V_0 + V_1 = V_2$ (3) ; (1) and (3) $\Rightarrow V_1 = \frac{m_1 - m_2}{m_1 + m_2}V_0$; $\Rightarrow \left\lvert\frac{V_1}{V_0}\right\rvert = \left\lvert\frac{m_1 - m_2}{m_1 + m_2}\right\rvert = \frac{11}{13} = k$	1.25
B.2.b	$\frac{K.E_1}{K.E_0} = \frac{V_1^2}{V_0^2} = k^2$**,** after the first collision, also $\frac{KE_2}{KE_1} = k^2$ after the second collision $\Rightarrow \frac{KE_2}{KE_0} = \left(k^2\right)^2 = k^4$, we demonstrate by recurrence that: $\frac{KE_n}{E_0} = k^{2n} \Rightarrow \frac{0.04}{2\times10^6} = k^{2n} \Rightarrow n = \frac{1}{2}\left(\frac{\ln 2.\ 10^{-8}}{\ln\frac{11}{13}}\right) = 53$ collisions	1.00