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الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ثلاث ساعات	

<u>This exam is formed of four exercises in four pages numbered from 1 to 4.</u>
<u>The use of a non-programmable calculator is recommended.</u>

<u>First exercise</u>: (8 points) Oscillation and rotation of a mechanical system

A rigid rod AB, of negligible mass and of length L = 2 m, may rotate, without friction, around a horizontal axis (Δ) perpendicular to the rod through its midpoint O. On this rod, and on opposite sides of O, two identical particles (S) and (S′), each of mass m = 100 g, may slide along AB.

Take: the gravitational acceleration on the Earth $g = 9.8\ m/s^2$;

for small angles: $\cos\theta = 1 - \frac{\theta^2}{2}$ and $\sin\theta = \theta$ in rad.

A – Oscillatory motion

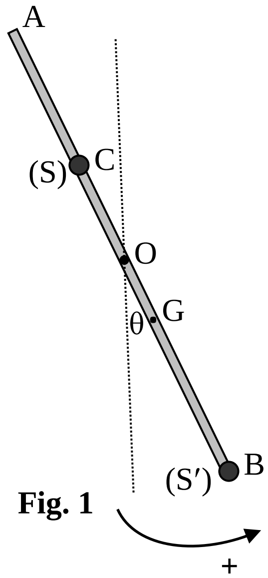


Fig. 1

The particle (S) is fixed on the rod at point C at a distance $OC = \frac{L}{4}$ and the particle (S′) is fixed at point B (Fig. 1). G is the center of gravity of the system (P) formed of the rod and the two particles. Let OG = a and I_0 be the moment of inertia of (P) with respect to the axis (Δ).

We shift (P) by a small angle θ_m, about (Δ), from its stable equilibrium position, in the positive direction as shown on the figure, and then released without initial velocity at the instant $t_0 = 0$; (P) thus oscillates, around the axis (Δ) with a proper period T. At an instant t, the angular abscissa of the compound pendulum, thus formed, is θ; (θ is the angle formed between the rod and the vertical passing through O), and its angular velocity is $\theta' = \frac{d\theta}{dt}$.We neglect all frictional forces and take the horizontal plane through O as a gravitational potential energy reference.

1) Show that $a = \frac{L}{8}$.

2) Show that $I_0 = \frac{5mL^2}{16}$.

3) Write, at an instant t, the expression of the mechanical energy of the system [Earth, (P)] in terms of I_0, m, a, g, θ and θ'.

4) Derive the second order differential equation in θ that describes the motion of (P).

5) Deduce, in terms of L and g, the expression of T. Calculate its value on the the Earth.

6) The system (P) oscillates now on the Moon. In this case, the proper period, for small oscillations, is T′. Compare, with justification, T′ and T.

B – Rotational motion

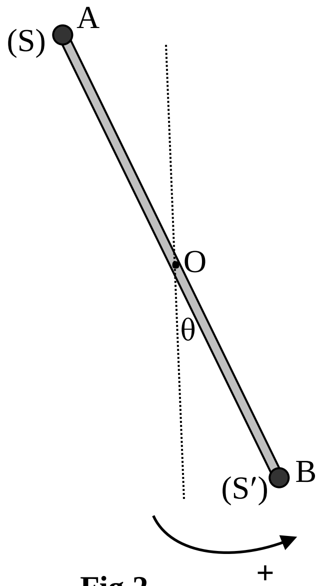


Fig.2

In this part, the particles (S) and (S′) are fixed at A and B respectively (Fig.2).

At the instant $t_0 = 0$, we launch the system (P′) thus formed, around (Δ) with an initial angular velocity $\theta'_0 = 2$ rad/s ; (P′) then turns, in the vertical plane around (Δ).

At an instant t, the angular abscissa of the rod, with respect to the vertical passing through O, is θ , and its angular velocity is $\theta' = \frac{d\theta}{dt}$. During rotation, (P′) is acted upon by a couple of forces of friction whose moment, with respect to (Δ) is $M = -h\ \theta'$, where h is a positive constant.

1) Give the name, at an instant t, of the couple and the forces acting on (P′).

2) Show that the resultant moment of the couple and of the forces, with respect to (Δ), is equal to the moment $M = -h\ \theta'$.

3) Show that the moment of inertia of (P′) about (Δ) is I = 0.2 kgm^2.

4) Using the theorem of angular momentum $\frac{d\sigma}{dt} = \Sigma M_{ext}$,

show that the differential equation in σ is written as :

$\frac{d\sigma}{dt} + \frac{h}{I}\sigma = 0$, σ is the angular momentum of (P′), about (Δ).

5) Verify that $\sigma = \sigma_0 e^{-\frac{h}{I}t}$ is a solution of the differential equation [σ_0 is the angular momentum of (P′), about (Δ), at the instant $t_0 = 0$].

6) The variation of σ as a function of time, is represented by the curve of figure 3. On this figure, we draw the tangent to the curve at point D at the instant $t_0 = 0$.

a) The curve of figure 3 is in agreement with the solution of the differential equation. Why?

b) Determine the value of h.

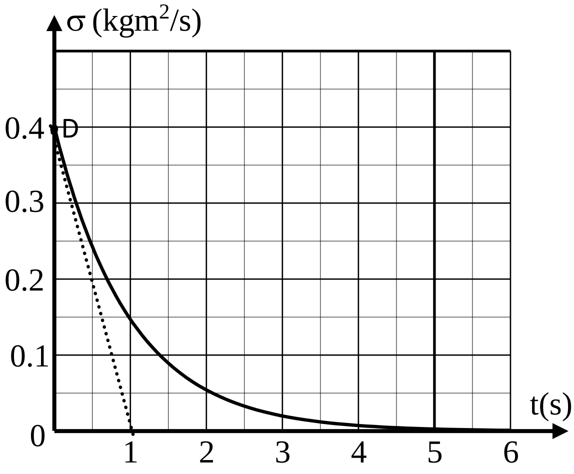


Fig.3

Second exercise: (6.5 points) **Charging and discharging of a capacitor**

We set up the circuit whose diagram is represented in figure 1, G is a generator of constant e.m.f E = 10 V and of negligible internal resistance, (C) is a capacitor, initially uncharged, of capacitance C = 1 F, (D) is a resistor of resistance R= 10 Ω, K is a switch and M is an electric motor whose axis is wrapped by a string of negligible mass and carrying a solid of mass m = 1 kg (Fig. 1). Take g = 10 m/s^2.

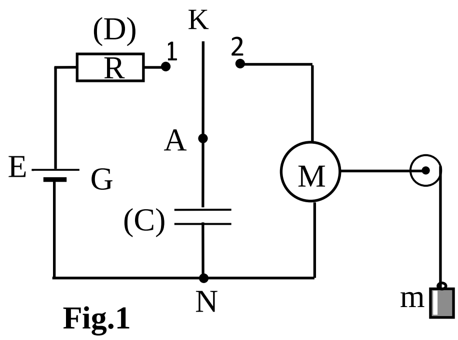


Fig.1

A – Charging of the capacitor

K is in position 1 at the instant $t_0 = 0$.

1) Determine the differential equation that describes the variation of the voltage $u_{AN} = u_C$ across the capacitor.

2) The solution of the differential equation is of the form:

$u_C = A + B\ e^{-\frac{t}{\tau}}$ where A, B and τ are constants.

Determine the expressions of A, B and τ in terms of E, R and C.

3) At the end of charging:

a) deduce the value of the voltage u_C ;

b) calculate, in J, the energy stored in the capacitor.

B – Discharging of the capacitor through the motor

The capacitor being totally charged, we turn the switch K to the position 2 at an instant taken as a new origin of time. During a time t_1, the solid is raised by height h = 1.5 m. At the instant t_1, the voltage across the capacitor is $u_C = u_1$.

The variation of the voltage u_C across the capacitor during discharging through the motor between the instants 0 and t_1 is represented by the curve of figure 2.

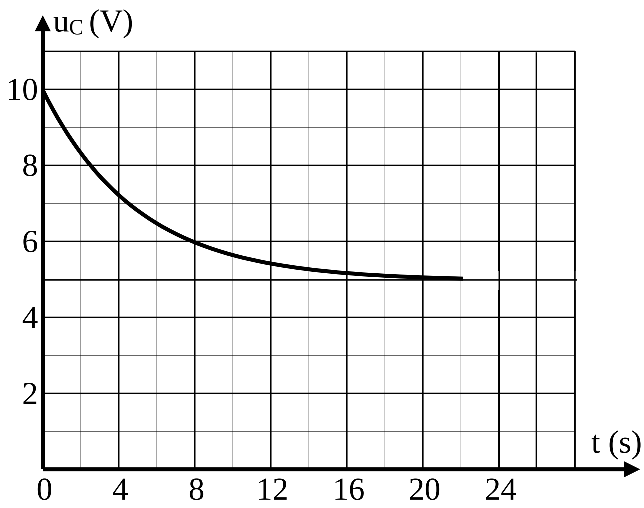


Fig.2

1) Referring to figure 2:

   a) give the value of t_1, at which the voltage u_C attains the minimum value u_1;
   b) give the value of the voltage u_1.
2) At the instant t_1, the capacitor still stores energy W_1.
   a) Tell why.
   b) Calculate the value of W_1 .
3) Assume that the energy yielded by the capacitor is received by the motor.
   a) Calculate the value of the energy W_2 yielded by the capacitor between the instants 0 and t_1.
   b) To what forms of energy is W_2 transformed?
   c) Determine the efficiency of the motor.

<u>Third exercise</u>: (8 points) **Electromagnetic oscillations**

An electric circuit is formed of a generator of constant e.m.f. E = 10 V and of negligible internal resistance, a capacitor, initially uncharged and of capacitance $C = 10^{-3}$F, a coil of inductance L = 0.1 H and of negligible resistance and a rheostat of variable resistance R.
In order to study the effect of R on the electric oscillations of an (R, L, C) circuit, we connect the circuit represented in figure (1).

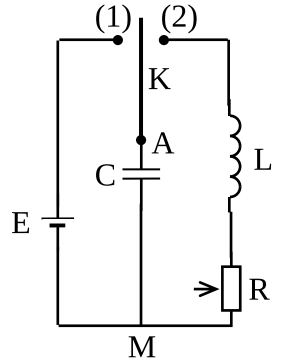


Fig.1

A – The switch is in position (1).
1) Give the name of the physical phenomenon that takes place in the electric circuit.
2) After closing the circuit for a sufficient time, specify the value of:
   a) the current;
   b) the voltage $u_{AM} = u_C$ across the capacitor;
   c) the electric energy W_{ele} stored in the capacitor.

B – The capacitor being totally charged, we turn the switch to position (2) at an instant $t_0 = 0$ taken as an origin of time.

I – The resistance of the rheostat is regulated at a value R = 0.
1) Derive the differential equation of the variation of $u_C = u_{AM}$ as a function of time.
2) The solution of the differential equation is of the form $u_C = E\cos(\frac{2\pi}{T_0}t)$.
   a) Determine, in terms of L and C, the expression of the proper period T_0 of the free electric oscillations that take place in the circuit.
   b) Calculate the value of T_0.
3) Express, as a function of time, the electric energy W_{ele} stored in the capacitor.
4) The electric energy W_{ele} is a periodic function of period T′. Write the relation between T′ and T_0.
5) Calculate the electric energy stored in the capacitor at the instant $t_0 = 0$.
6) Trace the shape of the graph of W_{ele} as a function of time.

II – The rheostat is regulated at a small resistance R.
The variation of the electric energy W_{ele} as a function of time is represented in figure (2).
Referring to this figure:
1) give the name of the type of the electric oscillations;
2) determine the value of the pseudo-period T of the electric oscillations;
3) justify that at the instants: 0 ; 31.5 ms ; 63 ms ; t_2 = 94.5 ms ; 126 ms, the total energy stored in the circuit is electric;

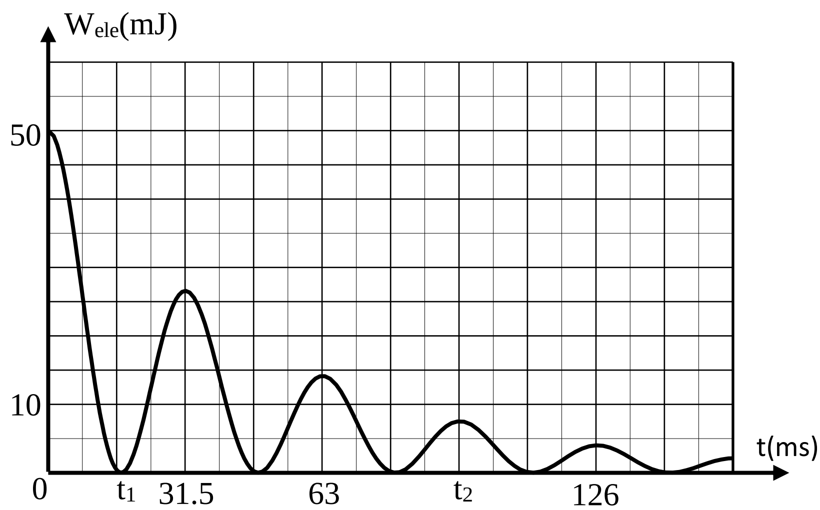


Fig.2

4) specify the form of the energy in the circuit at the instant t_1;
5) specify, between the instants $t_0 = 0$ and t = 31.5 ms, the time interval during which the:

– coil provides energy to the circuit;

– capacitor provides energy to the circuit;

6) calculate the energy dissipated in the rheostat between the instants $t_0 = 0$ and t_2.

III – What will happen if the resistance of the rheostat is very large?

__Fourth exercise__: (7.5 points) Spectrum of the hydrogen atom

Rydberg found in 1885 an empirical formula that gives the wavelengths of the lines of Balmer series; other series are discovered after that date.
An atom in an excited state n, passes to a lower energy state m, emits electromagnetic rays of wavelength λ , such that:

$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right),\ \lambda \text{ in meter and } R = 1.097\times10^{7}\text{m}^{-1}.$$

Given: Speed of light in vacuum $c = 2.998\times10^{8}$m/s;

Planck′s constant $h = 6.626\times10^{-34}$ J.s;
1 ev = 1.60×10^{-19}J.

1) Show that the energy E_n of the hydrogen atom, corresponding to an energy level n, can be expressed as $E_n = -\frac{hcR}{n^2}$.

2) Deduce that the energy E_n , expressed in eV, may be written in the form $E_n = -\frac{13.6}{n^2}$.

3) Calculate the value of the:
a) maximum energy of the hydrogen atom;
b) minimum energy of the hydrogen atom ;
c) energy of the hydrogen atom in the first excited state E_2 ;
d) energy of the atom in the second excited state E_3 .

4) Deduce that the energy of the atom is quantized.

5) Give three characteristics of a photon.

6) **a)** Define the ionization energy W_i of the hydrogen atom, found in the ground state.
b) Calculate the value of W_i.
c) Calculate the value of the wavelength of the radiation capable of producing this ionization.

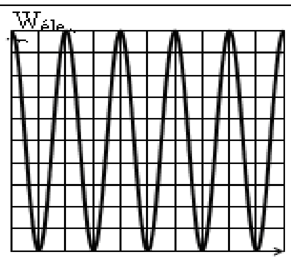
7) The Lyman series corresponds to the lines emitted by the excited hydrogen atom in a downward transition to the fundamental state.
a) Determine the shortest and the longest wavelengths of this series.
b) To what domain (visible, infrared, ultraviolet) does it belong?

8) **a)** Calculate the frequencies $\nu_{3\to1}$, $\nu_{2\to1}$, and $\nu_{3\to2}$ of the emitted photons corresponding respectively to the transitions $E_3 \rightarrow E_1$, $E_2 \rightarrow E_1$ and $E_3 \rightarrow E_2$ of the hydrogen atom.
b) Verify Ritz relation: $\nu_{3\to1} = \nu_{3\to2} + \nu_{2\to1}$.

First exercise : Oscillation and rotation of a mechanical system		8
Question	**Answer**	
A–1	$2ma = m\frac{L}{2} - m\frac{L}{4} = m\frac{L}{4} \Rightarrow a = \frac{L}{8}$.	½
A–2	$I_0 = m(\frac{L}{2})^2 + m(\frac{L}{4})^2 = \frac{5mL^2}{16}$.	½
A–3	$ME = KE + PE_g = \frac{1}{2} I_0\theta'^2 - 2mga\cos\theta$	¾
A–4	$\frac{dME}{dt} = 0 = I_0\theta'\theta'' + 2mga\theta'\sin\theta \Rightarrow I_0\theta'' + 2mga\,\theta = 0 \Rightarrow \theta'' + \frac{2mga}{I_0}\theta = 0$.	¾
A–5	The proper pulsation of the pendulum $\omega = \sqrt{\frac{2mga}{I_0}} \Rightarrow$ The period is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_0}{2mga}} = 2\pi\sqrt{\frac{5mL^2 \times 8}{16 \times 2mg \times L}} = 2\pi\sqrt{\frac{5L}{4g}} =$ $2\pi\sqrt{\frac{5 \times 2}{4 \times 9.8}} = 3.17s$.	1
A – 6	g(Moon) < g(Earth) $\Rightarrow$ T(Moon) > T(Earth).	½
B–1	The weight, the reaction of the axis and the couple of forces of friction.	¼
B – 2	The weight and the reaction of the axis meet the axis, their moment is zero, the resultant moment is: $\Sigma M = M_{couple} = -h\theta'$. $\Rightarrow \Sigma M = M = -h\theta'$.	½
B – 3	$I = 2\,m(\frac{L}{2})^2 = m\frac{L^2}{2} = \frac{0.1 \times 4}{2} = 0.2\ kgm^2$.	½
B – 4	$\frac{d\sigma}{dt} = \Sigma M_{ext} = M = -h\theta'$, and $\sigma = I\theta' \Rightarrow \frac{d\sigma}{dt} = -\frac{h}{I}\sigma$ $\Rightarrow \frac{d\sigma}{dt} + \frac{h}{I}\sigma = 0$.	¾
B – 5	$\frac{d\sigma}{dt} = -\frac{h}{I}\sigma_0 e^{-\frac{h}{I}t} \Rightarrow -\frac{h}{I}\sigma_0 e^{-\frac{h}{I}t} + \frac{h}{I}\sigma_0 e^{-\frac{h}{I}t} = 0$.	½
B – 6 – a	Because at $t = 0$, $\sigma_0 = I \times \theta'_0 = 0.2 \times 2 = 0.4\ kgm^2/s$. decreasing curve and as $t \to 5s$, $\sigma \to 0$.	½
B – 6 – b	$\frac{d\sigma}{dt} = -\frac{h}{I}\sigma_0 e^{-\frac{h}{I}t}$, at $t = 0$, $\frac{d\sigma}{dt} = -\frac{h}{I}\sigma_0 = -\frac{0.4}{1} \Rightarrow h = \frac{0.4 \times 0.2}{0.4} = 0.2$ S.I.	1

Second exercise : Charging and discharging of a capacitor		6 ½
Question	**Answer**	
A–1	$E = Ri + u_C = RC\dfrac{du_C}{dt} + u_C$	½
A–2	$\dfrac{du_C}{dt} = -\dfrac{B}{\tau}\, e^{-\frac{t}{\tau}} \Rightarrow E = RC(-\dfrac{B}{\tau}\, e^{-\frac{t}{\tau}}) + A + B\,e^{-\frac{t}{\tau}} \Rightarrow$ A= E and $RC(-\dfrac{B}{\tau})+B = 0$ $\Rightarrow \tau = RC$. for t = 0, $u_C = 0 = A + B \Rightarrow B = -A = -E$	1 ½
A–3–a	$u_C = E\,(1 - e^{-\frac{t}{RC}})$, for $t \to \infty$, $u_C \to E = 10$ V.	½
A–3–b	$W = ½\, C\, E^2 = ½\,(1)\,(100) = 50$ J.	½
B–1–a	$t_1 = 22$ s.	¼
B–1–b	$u_1 = 5$ V.	¼
B–2–a	because $u_C = u_1 = 5\ V \neq 0$.	¾
B–2–b	$W_1 = ½\, C(u_C)^2 = ½\,(1)\,(5)^2 = 12.5$ J.	½
B–3–a	$W_2 = W - W_1 = 50 - 12.5 = 37.5$ J.	½
B–3–b	Thermal and mechanical (kinetic)	¼
B–3–c	$r = \dfrac{mgh}{W_2} = \dfrac{1 \times 10 \times 1.5}{37.5} = 40$ %.	1

Third exercise : Electromagnetic Oscillations		8
Question	**Answer**	
A–1	Charging of the capacitor	¼

A–2–a–b–c	$i = 0$; $u_C = E = 10V$; $W_{ele} = ½CE^2 = ½(10^{-3})(100) = 0.05$ J.	¾
B–I–1	$u_C = u_{AM} = L\frac{di}{dt}$, $i = -C(u_C)' \Rightarrow \frac{di}{dt} = -C(u_C)'' \Rightarrow (u_C)'' + \frac{1}{LC}u_C = 0$	1
B–I–2–a	$(u_C)' = -\frac{2\pi}{T_0}E\sin\frac{2\pi}{T_0}t$, $(u_C)'' = -(\frac{2\pi}{T_0})^2E\cos\frac{2\pi}{T_0}t$, replace in the differential equation we get : $-(\frac{2\pi}{T_0})^2E\cos\frac{2\pi}{T_0}t + \frac{1}{LC}E\cos\frac{2\pi}{T_0} = 0 \Rightarrow (\frac{2\pi}{T_0})^2 = \frac{1}{LC} \Rightarrow T_0 = 2\pi\sqrt{LC}$	1
B–I–2–b	$T_0 = 2\pi\sqrt{10^{-4}}$ =0.0628 s = 62.8 ms.	¼
B–I–3	$W_{ele} = ½ C(u_C)^2 = ½ CE^2\cos^2(\frac{2\pi}{T_0}t) = 0.05\cos^2(100t)$.	½
B–I–4	$T' = T_0/2$.	¼
B–I–5	At $t_0 = 0$, $W_{ele} = 0.05$ J.	¼
B-I–6		½
B–II–1	free-damped electric oscillations.	¼
B –II–2	2T = 126 ms ; T = 63 ms.	½
B –II–3	At the instants :0 ; 31.5 ms ; 63 ms ; 94.5 ms ; 126 ms ; the electric energy is maximum $\Rightarrow$ u_C is max.$\Rightarrow i = C(u_C)' = 0 \Rightarrow$ magnetic energy $E_{mag} = ½ L(i)^2$ is zero$\Rightarrow$ E_{total} is electric.	3/4
B – II –4	Magnetic energy	¼
B –II –5	$0 < t < t_1$: W_{ele} decreases $\Rightarrow$ the capacitor gives energy to the circuit. $t_1 < t < 31.5$ ms : W_{ele} increases$\Rightarrow$ the coil gives energy to the circuit.	½
B –II –6	W(dissipated) = 50 – 7.5 = 42.5 mJ.	½
B – III	The electric energy is lost quickly in the resistor and the mode is not oscillatory	½

Fourth exercise : Spectrum of the hydrogen atom		**7 ½**
Question	**Answer**	
1	$E_n - E_m = \frac{hc}{\lambda} = hc \times R(\frac{1}{m^2} - \frac{1}{n^2}) \Rightarrow E_n = -\frac{hcR}{n^2}$	¾
2	$hcR = 6.626\times10^{-34} \times 2.998\times10^{8} \times 1.097\times10^{7}$ (in J) $= 21.79\times10^{-19}$ J = 13.6 eV$\Rightarrow$ $E_n = -\frac{13.6}{n^2}$ eV.	¾

3–a	as $n \to \infty$, $E_{max} \to 0$.	¼
3–b	as $n \to 1$; $E_{min} = -13.6$ ev	¼
3–c	$E_2 = -\frac{13.6}{2^2} = -3.4\text{eV}$	¼
3–d	E_3 for n = 3 $\Rightarrow E_3 = -1.51$ eV.	¼
4	Only certain values of E_n (–13.6; –3.4; –1.51; –0.85 ……) are allowed	¼
5	The photon: no mass, no charge, speed in vacuum is c, of energy hν.	¾
6–a	The ionization energy is the energy needed for the atom to absorb for it to release its electron without speed.	½
6–b	$W_i + (-13.6) = 0$; $W_i = 13.6$ eV.	½
6–c	$\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$ for $n \to \infty$ and m = 1, $\frac{1}{\lambda} = R = 1.097 \times 10^7 \Rightarrow \lambda = 0.911 \times 10^{-7}$ m.	½
7–a	$\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$; for m =1 and n = 2, we obtain $\lambda_{max} = 0.121 \times 10^{-6}$m for m=1 and $n \to \infty$, we obtain $\lambda_{min} = 0.091 \times 10^{-6}$m.	½
7–b	ultra-violet.	¼
8–a	$\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$. for m=1 and n=3, $\frac{1}{\lambda} = R(\frac{1}{1^2} - \frac{1}{3^2}) = \frac{8}{9}R = 0.975 \times 10^7$ $\nu = \frac{c}{\lambda} \Rightarrow$ $\nu_{3\to1} = 2.92 \times 10^{15}$ Hz. for m=1 and n=2 on a $\frac{1}{\lambda} = R(\frac{1}{1^2} - \frac{1}{2^2}) = \frac{3}{4}R = 0.82275 \times 10^7 \Rightarrow \nu_{2\to1} = 2.47 \times 10^{15}$ Hz. for m =2 and n=3 on a $\frac{1}{\lambda} = R(\frac{1}{2^2} - \frac{1}{3^2}) = \frac{5}{36}R = 0.15236 \times 10^7 \Rightarrow$ $\nu_{3\to2} = 0.46 \times 10^{15}$ Hz.	1 ¼
8–b	$\nu_{3\to1} = \nu_{3\to2} + \nu_{2\to1}$ is verified	½