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الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ثلاث ساعات	

This exam is formed of four exercises in four pages numbered from 1 to 4.
The use of non-programmable calculator is recommended

First Exercise: (7 ½ points)

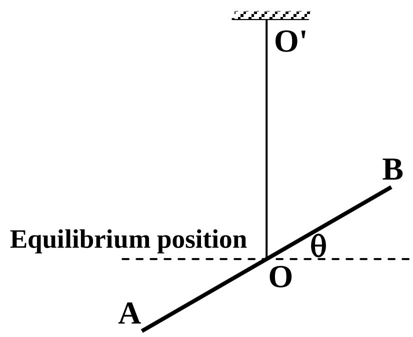
Torsion pendulum

The object of this exercise is to determine the moment of inertia I of a homogeneous rod AB with respect to an axis perpendicular to the rod at its midpoint and the torsion constant C of a wire OO' of negligible mass.

The rod has a mass M and a length AB = ℓ = 60 cm.

A torsion pendulum [P] is obtained by fixing the mid-point of AB to one end O of the wire while the other end O' is fixed to a support .

The rod is shifted, from its equilibrium position, by a small angle θ_m in the horizontal plane and it is released from rest at an instant $t_0 = 0$ .The rod thus may turn in the horizontal plane about an axis (Δ) passing through OO'.



At an instant t during motion, the angular abscissa of the rod is θ and its angular velocity is $\dot{\theta} = \frac{d\theta}{dt}$

The horizontal plane containing the rod is taken as a gravitational potential energy reference.

We neglect any force of friction and take $\pi^2 = 10$.

***A* – Theoretical study**

1) Give, at the instant t, the expression of the mechanical energy M.E of the system [(P), Earth] in terms of I, C, θ and $\dot{\theta}$.

2) ***a)*** Write the expression of M.E when $\theta = \theta_m$

b) Determine , in terms of C, θ_m and I, the expression of the angular speed of [P] as it passes through its equilibrium position .

3) Derive the second order differential equation in θ that governs the motion of [P].

4) Deduce that the motion of [P] is sinusoidal.

5) Determine the expression of the proper period T_1 of the pendulum in terms of I and C.

***B* – Experimental study**

1) By means of a stopwatch, we measure the duration t_1 of 20 oscillations and we obtain t_1 = 20 s. Determine the relation between I and C.

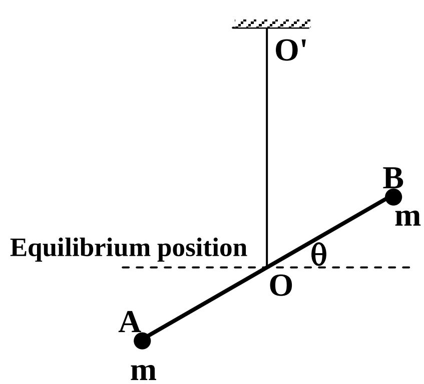
2) At each extremity of the rod we fixe a particle of mass m = 25g . We thus obtain a new torsion pendulum [P'] whose motion is also rotational sinusoidal of proper period T_2.

a) Determine the moment of inertia I' of the system (rod + particles) with respect to the axis (Δ) in terms of I, m, and ℓ .

b) Write down the expression of T_2 in terms of I , C , m and ℓ .

c) By means of a stopwatch, we measure the duration t_2 of 20 oscillations and we obtain t_2 = 40 s.

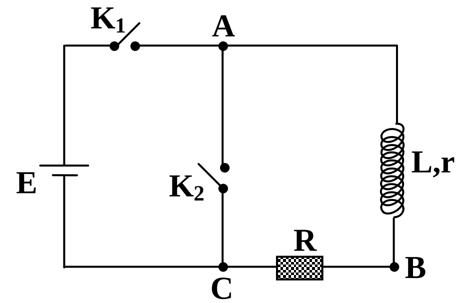
Find a new relation between I and C.



3) Calculate the values of I and C.

Second Exercise: (7 ½ points)

The phenomenon of self-induction



The set up represented by the adjacent figure consists of an ideal generator of emf E = 12 V, a coil of resistance r = 10 Ω and of inductance L = 40 mH, a resistor of resistance R = 40 Ω and two switches K_1 and K_2.

A – At the instant $t_0 = 0$, we close the switch K_1 and we leave K_2 open. At an instant t, the circuit carries a current i_1 in the transient state.

1) Derive the differential equation that governs the variation of i_1 as a function of time.

2) I_0 is the current in the steady state. Determine the expression of I_0 in terms of E, r and R and calculate its value.

3) The solution of the differential equation is of the form: $i_1 = I_0(1 - e^{\frac{-t}{\tau}})$.

a) Determine the expression of τ in terms of L, r and R and calculate its value.

b) Give the physical significance of τ.

4) **a)** Determine the expression of the self-induced emf e_1 as a function of time t.

b) Calculate the algebraic value of e_1 at the instant $t_0 = 0$.

B – After a few seconds, the steady state being reached, we open K_1 and we close K_2 at the same instant. We consider the instant of closing K_2 as a new origin of time $t_0 = 0$. The circuit (L, R, r) thus carries an induced current i_2 at an instant t.

1) Determine the direction of i_2.

2) Derive the differential equation that governs the variation of i_2 as a function of time.

3) Verify that $i_2 = I_0 e^{\frac{-t}{\tau}}$ is the solution of this differential equation.

4) Calculate the algebraic value of the self-induced emf e_2 at the instant $t_0 = 0$.

C – Compare e_1 and e_2 and deduce the role of the coil in each of the two previous circuits.

Third Exercise: (7 ½ points)

Characteristics of an (R, L, C) circuit

In order to determine the characteristics of an (R, L, C) circuit, we connect the circuit represented in figure 1. This circuit is formed of a resistor of resistance R= 650 Ω, a coil of inductance L and of negligible resistance and a capacitor of capacitance C, all connected in series across a function generator (LFG) delivering across its terminals a sinusoidal alternating voltage u_g of the form:

$u_g = u_{AM} = U_m \cos(2\pi f)t$.

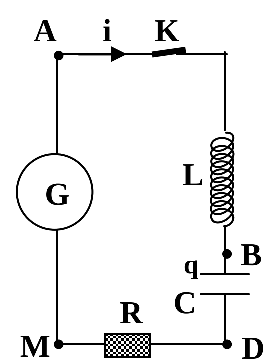


Fig.1

A –The frequency of the voltage u_G is adjusted on the value f_1 .
We display, on the screen of an oscilloscope, the variations , as a function of time, of the voltage u_{AM} across the generator on the channel (Y_1) and the voltage u_{DM} across the resistor on the channel (Y_2) .
The waveforms obtained are represented in figure 2.

Vertical sensitivity on both channels is: 2 V/div.

Horizontal sensitivity is: 0.1 ms /div.

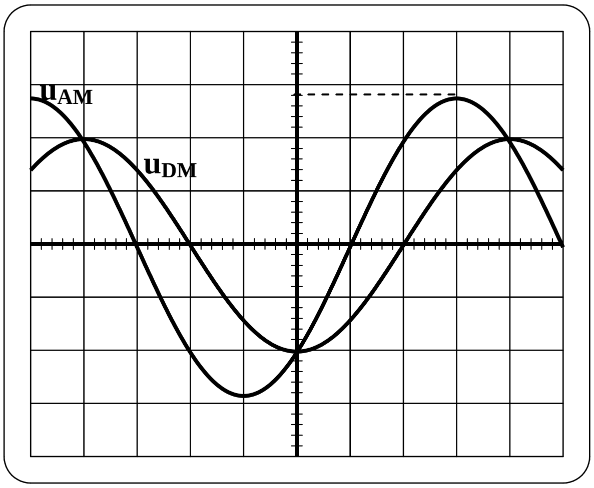


Fig.2

1) Redraw figure (1) showing on it the connections of the oscilloscope.

2) Referring to the waveforms, determine:

- ***a)*** The value of the frequency f_1.
- ***b)*** The absolute value of φ_1 the phase difference between u_{AM} and u_{DM}.

3) The current i carried by the circuit has the form:

$i = I_m \cos(2\pi f_1 t - \varphi_1)$.

- ***a)*** Write down the expressions of the voltages: u_{AB}, u_{BD} and u_{DM} as a function of time.
- ***b)*** The relation: $u_{AM} = u_{AB} + u_{BD} + u_{DM}$ is valid for any instant t . Show, by giving t a particular value, that:

$$\tan \varphi_1 = \frac{L(2\pi f_1) - \frac{1}{C(2\pi f_1)}}{R}$$

B – Starting from the value f_1, we decrease continuously the frequency f . We notice that, for f_0 = 500 Hz the circuit is the seat a of current resonance phenomenon.

Deduce from what preceded a relation between L, C and f_0.

C – We keep decreasing the frequency f . For a value f_2 of f we find that the phase difference between u_{AM} and u_{DM} is φ_2 such that $\varphi_2 = - \varphi_1$.

1) Determine the relation among f_1, f_2 and f_0 .

2) Deduce the value of f_2.

D – Deduce from what is preceded the values of L and C.

<u>Fourth Exercise</u>: (7 ½ points)

Energy levels of the hydrogen atom

The energies of the various levels of the hydrogen atom are given by the relation:

$E_n = -\frac{E_0}{n^2}$, where E_0 is a positive constant and n is a positive whole number.

Given:

Planck's constant $h = 6.62\times10^{-34}$ J.s; 1 eV = 1.6×10^{-19} J; 1 nm = 10^{-9} m.
Speed of light in vacuum: $c = 3\times10^{8}$ m/s

1) ***a)*** The energy of the hydrogen atom is quantized. What is meant by "*quantized energy*"?
   b) Explain why the absorption or emission spectrum of hydrogen consists of lines.

2) A hydrogen atom, initially excited, undergoes a downward transition from the energy level E_2 to the energy level E_1. It then emits the radiation of wavelength in vacuum: $\lambda_{2\to1} = 1.216\times10^{-7}$ m.
   Determine, in J, the value:
   a) of the constant E_0;
   b) of the ionization energy of the hydrogen atom taken in its ground state.

3) For hydrogen, we define several series that are named after the researchers who contributed in their study . Among these series we consider that of Balmer, which is characterized by the downward transitions from the energy level $E_p > E_2$ (p >2) to the energy level E_2 (n = 2).
   To each transition $p \to 2$ corresponds a line of wave $\lambda_{p\to2}$.
   a) Show that $\lambda_{p\to2}$, expressed in nm, is given by the relation:. $\frac{1}{\lambda_{p\to2}} = 1.096\times10^{-2}\left[\frac{1}{4}-\frac{1}{p^2}\right]$
   b) The analysis of the emission spectrum of the hydrogen atom shows four visible lines.
   We consider the three lines H_α , H_β and H_γ of respective wavelengths in vacuum are
   λ_α = 656.28 nm ; λ_β = 486.13 nm and λ_γ = 434.05 nm.
   To which transition does each of these radiations correspond?
   c) Show that the wavelengths of the corresponding radiations tend ,when $p \to \infty$, towards a limit λ_0. whose value is to be calculated.

4) Balmer, in 1885, knew only the lines of the hydrogen atom that belong to the visible spectrum. He wrote the formula**:** $\lambda = K\frac{p^2}{p^2-4}$, where K is a positive constant and p a positive whole number.
   Determine the value of K using the numerical values and compare its value with that of λ_0.

First exercise (7.5 points)

A – 1) $ME = KE + P.E_g + PE_e = \frac{1}{2}I\theta'^2 + 0 + \frac{1}{2}C\theta^2$ **(¾)**

2) a) for max. deviation, $\theta = \theta_m$ and $\theta' = 0$.

$E_m = \frac{1}{2}C\theta^2_m$ **(½)**

b) At equilibrium position, $E_m = \frac{1}{2} I\theta'^2_m$

$\Rightarrow \theta' = \pm\theta_m\sqrt{\frac{C}{I}}$ **(¾)**

3) M.E is conserved since no friction thus the derivative of M.E w.r.t time is zero

$I\theta'\theta'' + C\theta\theta' = 0$, $\theta' \neq 0$ thus $\theta'' + \frac{C}{I}\theta = 0$ **(¾)**

4) Equation has the form $\theta'' + \omega^2\theta = 0$

It has a sinusoidal solution where $\omega^2 = \frac{C}{I}$: **(¼)**

5) $\omega^2 = \frac{C}{I}$

$\omega = \frac{2\pi}{T}$ thus $T_1 = 2\pi\sqrt{\frac{I}{C}}$ **(¾)**

B – 1) $t_1 = 20T_1 = 20s$ thus $T_1 = 1 = 2\pi\sqrt{\frac{I}{C}}$

and $40\frac{I}{C} = 1$ then $C = 40\,I$ **(1)**

2) a) $I' = I + 2m(\frac{\ell}{2})^2 = I + m\frac{\ell^2}{2} = I + 0.0045$ **(¾)**

b) Same law of motion thus $T_2 = 2\pi\sqrt{\frac{I'}{C}} = 2\pi\sqrt{\frac{I + \frac{m\ell^2}{2}}{C}}$ **(½)**

c) $T_2 = 2$ thus $10\frac{I'}{C} = 1$ or $C = 10I' = 10(I + 0.0045) = 10\,I + 0.045$ **(¾)**

3) $C = 40\,I = 10I + 0.045 \Rightarrow I = 1.5\times10^{-3}$ kg.m^2 and $C = 0.06$ N×m **(¾)**

Second exercise (7.5 points)

A – 1) $E = ri_1 + L\frac{di_1}{dt} + Ri_1 \Rightarrow E = (r+R)\,i_1 + L\frac{di_1}{dt}$. **(½)**

2) When the steady state mode is established, i_1 becomes constant and $\frac{di_1}{dt} = 0$;

the current is then I_0 such that: $E = (r+R)\,I_0 \Rightarrow I_0 = \frac{E}{R+r}$.

$I_0 = \frac{12}{40+10} = 0.24$ A **(1)**

3) a) $\frac{di_1}{dt} = I_0/\tau\,(e^{\frac{-t}{\tau}}) \Rightarrow E = (r+R)\,I_0(1 - e^{\frac{-t}{\tau}}) + L\,I_0/\tau\,(e^{\frac{-t}{\tau}})$

$\Rightarrow L/\tau = (r+R) \Rightarrow \tau = \frac{L}{R+r} = \frac{0.04}{50} = 0.8$ ms. **(1)**

b) The time constant characterizes the duration of the growth of the current in a (R+r, L) component **(¼)**

4) a) $e_1 = -L\frac{di_1}{dt} = -L\,I_0/\tau\,(e^{\frac{-t}{\tau}}) = -E\,e^{\frac{-t}{\tau}}$. **(½)**

b) For $t = 0$, $e_1 = -E = -12$ V. **(¼)**

B – 1) According to Lenz law, the coil carries a current i_2 of the same direction as that of i_1. **(½)**

2) $u_{AC} = u_{AB} + u_{BC} \Rightarrow 0 = ri_2 + L\frac{di_2}{dt} + Ri_2 \Rightarrow L\frac{di_2}{dt} + (R+r)\,i_2 = 0$ **(¾)**

3) $\frac{di_2}{dt} = -I_0/\tau\,e^{\frac{-t}{\tau}} \Rightarrow -L\,I_0/\tau\,e^{\frac{-t}{\tau}} + (R+r)\,I_0\,e^{\frac{-t}{\tau}} = 0$ **(1)**

4) $e_2 = -L\frac{di_2}{dt} = -L(-I_0/\tau\,e^{\frac{-t}{\tau}}) = E\,e^{\frac{-t}{\tau}}$.

At $t = 0$, $e_2 = E = 12$ V. **(¾)**

C – $e_1 = -e_2$.

When K_1 is closed, the self-induced emf opposes the growth of the current in the circuit

$\Rightarrow e_1 < 0$ (the coil plays the role of a generator in opposition).

When K_2 is closed, the self-induced emf opposes the decay of the current in the circuit

$\Rightarrow e_2 > 0$ (the coil plays the role of a generator). **(1)**

Third exercise (7.5 points)

A – 1) Connections of the oscilloscope. **(¼)**

2) a) $T_1 \rightarrow$ 8 div $\Rightarrow T_1 = 0{,}8$ ms

$f_1 = 1/T_1 = 1/\,0.8\times 10^{-3} = 1250$ Hz **(½)**

b) $|\varphi_1| = 2\pi\, 1/8 = \pi/4$ rad. **(¼)**

3) a) $i = I_m\cos(2\pi f_1 t - \varphi_1)$; $u_{AB} = L\, di/dt = -LI_m(2\pi f_1)\sin(2\pi f_1 t - \varphi_1)$

$u_C = 1/C \int i\, dt = I_m/C \int\cos(2\pi f_1 t - \varphi_1)\, dt$

$u_C = (I_m / C.2\pi f_1)\sin(2\pi f_1 t - \varphi_1)$

$u_R = Ri = R I_m\cos(2\pi f_1 t - \varphi_1)$ **(1)**

b) $U_m\cos 2\pi f_1 t = R I_m\cos(2\pi f_1 t - \varphi_1) + (I_m / C.2\pi f_1)\sin(2\pi f_1 t - \varphi_1) -$

$LI_m(2\pi f_1)\sin(2\pi f_1 t - \varphi_1)$

$2\pi f_1 t = \pi/2 \Rightarrow 0 = RI_m\sin\varphi_1 + (I_m / C.2\pi f_1)\cos\varphi_1 - L I_m(2\pi f_1)\cos\varphi_1$

$\Rightarrow R\sin\varphi_1 = [L(2\pi f_1) - 1/(C(2\pi f_1))]I_m\cos\varphi_1$

$$tg\varphi_1 = \frac{L(2\pi f_1) - \dfrac{1}{C(2\pi f_1)}}{R} \quad \textbf{(¾)}$$

B – Current resonance $\Rightarrow \varphi = 0 \Rightarrow tg\varphi = 0 \Rightarrow L2\pi f_0 - 1/C(2\pi f_0) = 0$

$\Rightarrow LC4\pi^2 f_0^2 = 1.$ **(¾)**

C – 1) $tg\varphi_1 = -\,tg\,\varphi_2 \Rightarrow \dfrac{L(2\pi f_1) - \dfrac{1}{C(2\pi f_1)}}{R} = \dfrac{\dfrac{1}{C2\pi f_2} - L2\pi f_2}{R}$

$\Rightarrow L2\pi f_1 + L2\pi f_2 = 1/C\,[1/(2\pi f_1) + 1/(2\pi f_2)]$

$LC = 1/4\pi^2 f_1 f_2 = 1/4\pi^2 f_0^2 \Rightarrow f_0^2 = f_1 f_2$ **(1½)**

2) $f_2 = (500^2)/1250 = 250000/1250 = 200$ Hz **(½)**

D – $\varphi_1 = \pi/4 \Rightarrow L2\pi(1250) - 1/(C\times 2\pi\times 1250) = 650$

$LC = 1/(4\pi^2 500^2) = 10^{-7} \Rightarrow LC\times 4\pi^2\times 1250^2 - 1 = 650\times C\times 2\pi\times 1250$

$\Rightarrow C = 5.25/(650\times 2\pi\times 1250) = 10^{-6}$ F $= 1\ \mu$F

$L = 10^{-7}/10^{-6} = 10^{-1}$ H $= 0.1$ H **(2)**

Fourth exercise (7.5 points)

1) a) The energies of the hydrogen atom can take only well defined values (discrete) **(½)**

b) For an electronic transition p $\rightarrow$ n the emitted photon (or absorbed) has a wavelength:

$\lambda_{p,n} = \dfrac{hc}{E_p - E_n}$. As E_p and E_N are quantized then $(E_p - E_N)$ is quantized too; which means that the $\lambda_{p,n}$ has a well determined value, which corresponds to a line.; **(1)**

1) a) $E_2 = -\dfrac{E_o}{4}$ and $E_1 = E_o \Rightarrow E_2 - E_1 = \dfrac{3E_0}{4} = \dfrac{hc}{\lambda_{2,1}}$

$$\Rightarrow E_0 = \frac{4\times 6.62\times 10^{-34}\times 3\times 10^8}{3\times 1.216\times 10^{-7}} = 2.177\times 10^{-18} \quad \textbf{(1½)}$$

b) $E_i = E_\infty - E_1 = E_0 = 2.177\times 10^{-18}$ J. **(½)**

3) a)

$$E_p - E_2 = -\frac{E_0}{p^2} + \frac{E_0}{4} = \frac{hc}{\lambda_{p,2}} \Rightarrow \frac{1}{\lambda_{p,2}} = \frac{E_0}{hc}\left(\frac{1}{4} - \frac{1}{p^2}\right)$$

$$= \frac{2.177\times 10^{-18}\times 10^{-9}}{6.62\times 10^{-34}\times 3\times 10^8}\left(\frac{1}{4} - \frac{1}{p^2}\right) = 1.096\times 10^{-2}\left(\frac{1}{4} - \frac{1}{p^2}\right) \quad \textbf{(1½)}$$

b) $\lambda_\alpha = 656.28$ nm $\Rightarrow$ p= 3, then it is the downward transition $3 \rightarrow 2$.

λ_β ; $4 \rightarrow 2$ and λ_γ ; $5 \rightarrow 2$. **(¾)**

c) when $p \rightarrow \infty \Rightarrow \lambda \rightarrow \lambda_0 = \dfrac{4}{1.096\times 10^{-2}} = 364.96$ nm **(½)**

4) For $\lambda_\alpha = 656.28$ nm, p = 3 ; $K = \lambda\dfrac{p^2 - 4}{p^2} = 364.6$ nm, $K \cong \lambda_0$ **(1¼)**