

الاسم:
الرقم:مسابقة في مادة الفيزياء
المدة: ثلاث ساعات

***This exam is formed of four exercises in 4 pages
numbered from 1 to 4.***

The use of a non-programmable calculator is recommended

First exercise (7 pts.) Determination of the characteristics of a coil

In order to determine the inductance L and the resistance r of a coil, we connect it in series with a capacitor of capacitance $C = 160 \mu\text{F}$ across the terminals of a low frequency generator (LFG) delivering an alternating sinusoidal voltage

$$u_g = u_{AD} = 20\sin(100\pi t) \quad (u \text{ in V, } t \text{ in s}) \quad (\text{figure 1}).$$

The circuit thus carries an alternating sinusoidal current i .

An oscilloscope is connected so as to display the voltage $u_g = u_{AD}$ on the channel Y_A , and the voltage $u_{\text{coil}} = u_{BD}$ on the channel Y_B .

We see on the screen of the oscilloscope a display of the waveforms represented in figure 2.

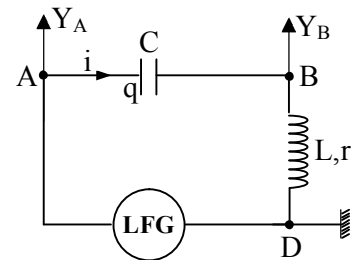


Figure 1

- 1) Knowing that the vertical sensitivity S_V is the same on both channels, calculate its value.
- 2) Calculate the phase difference between u_{AD} and u_{BD} .
Which of them lags behind the other?
- 3) Deduce the expression of the voltage u_{BD} across the terminals of the coil as a function of time.
- 4) Applying the law of addition of voltages, and giving the time t two particular values, verify that the voltage u_{AB} may be written as

$$u_C = u_{AB} = 20\sin\left(100\pi t - \frac{\pi}{3}\right) \quad (u_C \text{ in V, } t \text{ in s}).$$
- 5) Using the relation between the current i and the voltage u_C , determine the expression of i as a function of time.
- 6) a) Give the expression of the voltage u_{BD} across the terminals of the coil as a function of i .
b) Calculate r and L by giving t two particular values.
- 7) In order to verify the preceding calculated values of L and r , we proceed in the following way:
 - we measure the average power consumed in the circuit, for $\omega = 100\pi \text{ rad/s}$ and we obtain 8.66 W
 - we keep the maximum value of u_g constant but we vary its frequency f ; for $f = 71 \text{ Hz}$, the effective value of the current in the circuit is maximum.

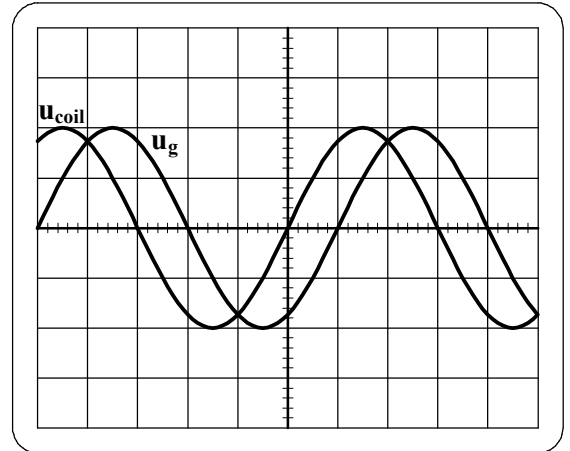


Figure 2

Determine the values of r and L .

Second exercise : (6 ½ pts)**Atomic nucleus**

The object of this exercise is to compare the values of physical quantities characterizing the stability of different nuclei and to verify that, during nuclear reactions, certain nuclei are transformed into more stable nuclei with the liberation of energy.

Numerical data:

Mass of a neutron: $m_n = 1.0087 \text{ u}$; mass of a proton : $m_p = 1.0073 \text{ u}$;
mass of an electron : $m_e = 0.00055 \text{ u}$; $1 \text{ u} = 931.5 \text{ MeV} / c^2$.

I – Stability of atomic nuclei

Consider the table below that shows some physical quantities associated with certain nuclei.

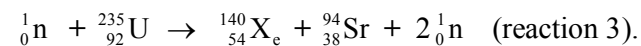
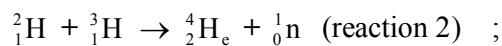
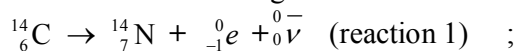
Nucleus	${}^2_1\text{H}$	${}^3_1\text{H}$	${}^4_2\text{He}$	${}^{14}_6\text{C}$	${}^{14}_7\text{N}$	${}^{94}_{38}\text{Sr}$	${}^{140}_{54}\text{Xe}$	${}^{235}_{92}\text{U}$
Mass (u)	2.0136	3.0155	4.0015	14.0065	14.0031	93.8945	139.892	234.9935
Binding energy E_b (MeV)	2.23	8.57	28.41	99.54	101.44	810.50	1164.75	
Binding energy per nucleon $\frac{E_b}{A}$ (MeV/nucleon)	1.11		7.10		7.25	8.62		

- 1) *a)* Define the binding energy of a nucleus.
- b)* Write the expression of the binding energy E_b of a nucleus ${}^A_Z\text{X}$ as a function of Z , A , m_p , m_n , m_X (the mass of the nucleus ${}^A_Z\text{X}$) and the speed of light in vacuum c .
- c)* Calculate , in MeV, the binding energy of the uranium 235 nucleus.
- d)* Complete the table by calculating the missing values of $\frac{E_b}{A}$.
- e)* Give the name of the most stable nucleus in the above table. Justify your answer.
- 2) Each of the considered nuclei in the table belongs to one of the three groups given by:
 $A < 20$; $20 < A < 190$; $A > 190$.

Referring to the completed table, trace the shape of the curve representing the variation of $\frac{E_b}{A}$ as a function of A . Specify on the figure the three mentioned groups.

II – Nuclear reactions and stability of the nuclei

Consider the following three nuclear reactions:



- 1) Indicate the type of each nuclear reaction (fission, radioactivity or fusion).
- 2) *a)* Show that each of the above nuclear reactions liberates energy.
- b)* Referring to the above table, verify that in each of these nuclear reactions, each of the produced nuclei is more stable than the initial nuclei.

Third exercise (6 ½ pts.) Index of refraction of atmospheric air

The index of refraction of pure air is supposed to be equal to 1.

Atmospheric air is not pure, it is polluted; it contains carbon dioxide.

The index of refraction n of polluted air is given by the relation:

$$n = 1 + 1.55 \times 10^{-6} y \quad \text{where } y \% \text{ represents the percentage of carbon dioxide in air.}$$

In order to determine the value of y , we use Young's double-slit apparatus of interference.

The two slits F_1 and F_2 , separated by a distance a , are illuminated with a laser beam of wavelength $\lambda = 0.633 \mu\text{m}$ in pure air.

The beam falls normally on the plane (P) that contains the slits.

We observe interference fringes on a screen (E) parallel to (P) found at a distance D from this plane.

Point O is the foot of the orthogonal projection of I, the mid point of F_1F_2 on the plane (E) (figure 1).

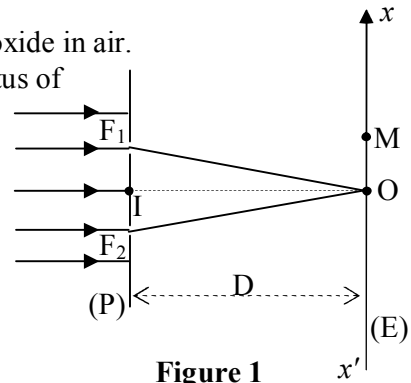


Figure 1

I – Interference in pure air

Recall that for a point M of the screen where $OM = x$, the optical path difference $\delta = MF_2 - MF_1$ is given

$$\text{by the relation } \delta = \frac{ax}{D}.$$

1) O is the center of the central bright fringe. Why?

2) M is the center of the bright fringe of order k .

a) Give the expression of δ in terms of k and λ .

b) Deduce the expression of the interfringe distance i in terms of λ , D and a .

3) M is the point of (E) so that $MF_2 - MF_1 = 1.266 \mu\text{m}$.

a) Specify the nature and the order of the fringe whose center is at M. Justify your answer.

b) Express x in terms of i .

II- Interference in polluted air

We intend to measure the index of refraction n of air polluted with carbon dioxide. In Young's double-slit apparatus used, we consider that the beam issued from F_2 propagates in pure air whereas the beam issued from F_1 propagates a distance $\ell = 50 \text{ cm}$ in polluted air and the rest of its path in pure air (figure 2).

We observe, in this case, that the system of interference fringes is displaced upwards.

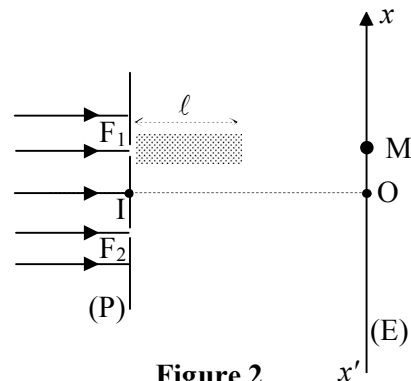


Figure 2

1- Knowing that v is the speed of light issued from F_1 in polluted air, give the expression of the time t that light needs to cover the distance ℓ in polluted air in terms of v and ℓ .

2- Knowing that c is the speed of light in pure air, determine the expression of the distance d that light issued from F_2 would cover in pure air during the same time t in terms of ℓ and n .

3- Give the expression of the increase in the optical path due to the passage in polluted air in terms of ℓ and n .

4- The new expression of the optical path difference is then: $\delta' = MF_2 - MF_1 = \frac{ax}{D} - \ell(n - 1)$

a) Knowing that the center of the central bright fringe is displaced up to occupy the position that was occupied by the center of the bright fringe of order 2, the interfringe distance being the same, determine the expression that gives n in terms of ℓ and λ .

b) Show that the value of n is 1.0000025.

5- a) The index n being given by: $n = 1 + 1.55 \times 10^{-6} y$, calculate the value of y .

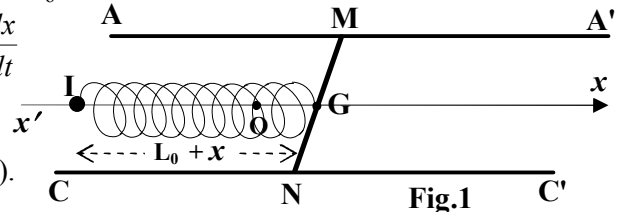
b) Air polluted with carbon dioxide becomes harmful when $y \geq 0.5$.

Is this polluted air harmful? Why?

Fourth exercise (7 ½ pts) Free mechanical oscillations

A horizontal elastic pendulum is formed of a homogeneous metallic rod MN of mass $m = 0.5 \text{ kg}$ and of length ℓ , and a spring of un-jointed turns of negligible mass having a stiffness $k = 50 \text{ N/m}$. The length of the spring, when free, is L_0 . One of the ends of this spring is connected at I to a fixed support while the other end is connected to the midpoint G of the rod. This rod may slide without friction along the metallic rails AA' and CC', that are horizontal and parallel to the axis $x'x$ of the spring; during sliding, the rod remains perpendicular to the rails, and G moves on the axis $x'x$. We move the rod, from its equilibrium position, kept parallel to itself, by 5cm in the positive direction and then we release it without initial velocity at the instant $t_0 = 0$.

At the instant t , the abscissa of G is $x = \overline{OG}$ and $v = \frac{dx}{dt}$ is the algebraic measure of its velocity; the origin of abscissa, O, corresponds to the position of G at equilibrium when the length of the spring is L_0 (figure 1).

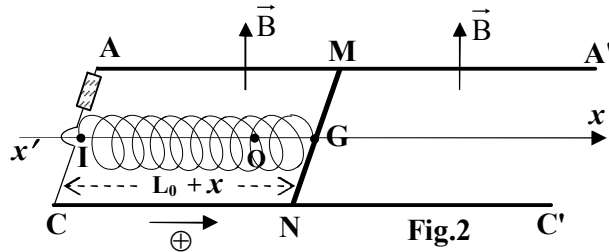


I – Free un-damped oscillations

- 1) Write, at the instant t , the expression of the mechanical energy M.E of the system (pendulum, Earth) in terms of m , x , k and v taking the horizontal plane through G as a gravitational potential energy reference.
- 2) Derive the second order differential equation in x that describes the motion of G.
- 3) The solution of this differential equation is given by the expression: $x = X_m \cos(\omega t + \varphi)$ where X_m is the amplitude of oscillations. Determine the values of ω , X_m and φ .

II –Free damped oscillations

The system formed of the pendulum and the rails is placed within a uniform magnetic field \vec{B} , perpendicular to the plane of the rails (figure 2).



We connect between A and C a resistor of convenient resistance; the resistance of the whole circuit is then R .

After shifting the rod by 5 cm in the positive direction, we release it from rest at the instant $t_0 = 0$. An induced current i passes in the circuit.

The horizontal pendulum performs few oscillations then comes to rest within an interval of time t_1 .

1) During motion, an induced electromotive force e appears across the ends M and N of the rod.

Explain why.

2) a) Determine, at the instant t , the expression of the magnetic flux through the surface limited by the circuit AMNC in terms of B , L_0 , x and ℓ , taking into account the arbitrary positive direction chosen in figure 2.

b) Deduce the expression of the induced emf e in terms of B , ℓ and v .

c) Determine the expression of i in terms of B , R , ℓ and v .

d) Specify the direction of the current induced when the rod is moving in the positive direction.

3) a) Interpret the damping of the oscillations and the stopping of the rod.

b) Calculate the mechanical energy of the oscillator at the instant $t_0 = 0$.

c) Deduce the value of the energy dissipated in the circuit between the instants $t_0 = 0$ and t_1 .

d) In what form is this energy dissipated?

Solution

First exercise : (7 pts)

1) The maximum voltage across the terminals of the generator corresponds to 2 div

$$\Rightarrow S_V = \frac{20 \text{ V}}{2 \text{ div}} = 10 \text{ V/div} \quad \left(\frac{1}{2} \text{ pt}\right)$$

2) The phase difference extends over 1 div and the period over 6 div

$$\varphi = \frac{(1)(2\pi)}{6} = \frac{\pi}{3} \text{ rad.} \quad u_g \text{ lags } u_{\text{coil}} \quad \left(\frac{3}{4} \text{ pt}\right)$$

3) $(U_m)_{\text{coil}} = 20 \text{ V}$, $\omega = 100\pi$ and u_{coil} leads u_g by $\frac{\pi}{3}$ rad $\Rightarrow u_{\text{coil}} = 20 \sin(100\pi t + \frac{\pi}{3})$

(1 pt)

4) $u_{AD} = u_{AB} + u_{BD}$. Let $u_{AB} = A \sin(100\pi t + \varphi)$.

$$20 \sin(100\pi t) = A \sin(100\pi t + \varphi) + 20 \sin(100\pi t + \frac{\pi}{3})$$

$$\text{For } t = 0, \text{ we get: } 0 = A \sin\varphi + 20 \frac{\sqrt{3}}{2}; \quad A \sin\varphi = -10\sqrt{3}$$

$$\text{For } 100\pi t = \frac{\pi}{2}, \text{ we get: } 20 = A \cos\varphi + 10; \quad A \cos\varphi = 10$$

$$\Rightarrow \varphi = -\frac{\pi}{3} \text{ and } A = 20. \Rightarrow u_{AB} = 20 \sin(100\pi t - \frac{\pi}{3}) \quad \left(\frac{3}{4} \text{ pt}\right)$$

5) $i = C \frac{du_C}{dt}$; $i = 160 \times 10^{-6} [20 \times 100\pi \cos(100\pi t - \frac{\pi}{3})] = 1 \cos(100\pi t - \frac{\pi}{3})$

$$= \sin(100\pi t + \frac{\pi}{6}) \quad \left(\frac{1}{2} \text{ pt}\right)$$

6) $u_{\text{coil}} = ri + L \frac{di}{dt}$; $20 \sin(100\pi t + \frac{\pi}{3}) = r \sin(100\pi t + \frac{\pi}{6}) + L(100\pi) [\cos(100\pi t + \frac{\pi}{6})]$

$$\text{For } t = 0, \text{ we obtain: } 20 \frac{\sqrt{3}}{2} = r(0.5) + 100\pi L \frac{\sqrt{3}}{2}$$

$$\text{For } 100\pi t = \pi/2, \text{ we obtain: } 10 = r \frac{\sqrt{3}}{2} - 50\pi L$$

$$\Rightarrow L = \frac{1}{10\pi} = 0.032 \text{ H and } r = 10\sqrt{3} \Omega. \quad \left(\frac{1}{2} \text{ pt}\right)$$

7) The electric power is consumed only in the resistor of the coil :

$$P = r (I_{\text{eff}})^2 = 8.66 = \left(\frac{1}{\sqrt{2}}\right)^2 r \Rightarrow r = 17.3 \Omega = 10\sqrt{3} \Omega. \quad \left(\frac{1}{2} \text{ pt}\right)$$

The observed phenomena is the current resonance. In this case we have :

$$LC\omega^2 = 1; \text{ or } L = \frac{1}{C\omega^2} = \frac{10^6}{160(142\pi)^2} = 0.032 \text{ H.} \quad \left(\frac{1}{2} \text{ pt}\right)$$

Second exercise : (6 ½ pts)

I - 1- a) The binding energy of a nucleus is the minimum energy needed in order to break the nucleus into its nucleons (½ pt)

b) $E_b = [Zm_p + (A-Z)m_n - m_X] c^2$ (½ pt)

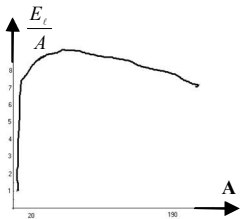
c) $\Delta m = Zm_p + (A-Z)m_n - m_X = 92 \times 1.0073 + 143 \times 1.0087 - 234.9935 = 1.9222 \text{ u}$
 $E_b = 1.9222 \times 931.5 \text{ Mev} = 1790.53 \text{ Mev.}$ (1 pt)

2- a) Table (½ pt)

^2_1H	^3_1H	^4_2He	$^{14}_6\text{C}$	$^{14}_7\text{N}$	$^{94}_{38}\text{Sr}$	$^{140}_{54}\text{Xe}$	$^{235}_{92}\text{U}$
1.11	2.8 6	7.10	7.11	7.2 5	8.62	8.32	7.62

b) The nucleus that has greater binding energy per nucleon is more stable \Rightarrow strontium is the most stable nucleus (½ pt)

c) Shape of the curve (½ pt)



II- 1) Reaction (1) : radioactivity ; reaction (2) : fusion ; reaction (3) : fission. (¾ pt)

2) a) For the radioactivity reaction $\Delta M = M_{\text{before}} - M_{\text{after}} = 14.0065 - (14.0031 + 0.00055) = 0.00285 \text{ u}$
 For the fusion reaction $\Delta M = M_{\text{before}} - M_{\text{after}} = 2.0136 + 3.0155 - 4.0015 - 1.0087 = 0.0189 \text{ u}$
 For the fission reaction $\Delta M = M_{\text{before}} - M_{\text{after}} = 0.1983 \text{ u}.$

In the 3 reactions there is a mass defect \Rightarrow The 3 reactions liberates energy. (1 ½ pt)

- b) - The $^{14}_7\text{N}$ nucleus is more stable than the $^{14}_6\text{C}$ nucleus.
 - The ^4_2He nucleus is more stable than the nuclei ^2_1H and ^3_1H .
 - The $^{140}_{54}\text{Xe}$ nucleus and the nucleus $^{94}_{38}\text{Sr}$ are more stable than the $^{235}_{92}\text{U}$ nucleus. (¾ pt)

Third exercise (6 ½ pts)

I – 1) The point O is characterized by $\delta = 0$, So a bright fringe is formed at O. $(\frac{1}{2} \text{ pt})$

2) a) $\delta = k \lambda \quad (\frac{1}{4} \text{ pt})$

b) $\delta = \frac{ax}{D} = k \lambda \Rightarrow x_k = \frac{k\lambda D}{a}$ and $x_{k+1} = \frac{(k+1)\lambda D}{a} \Rightarrow i = x_{k+1} - x_k = \frac{\lambda D}{a}$ **(1 pt)**

3) a) $\frac{MF_2 - MF_1}{\lambda} = 2$ is of the form $\delta = k \lambda$ such that $k = 2$; therefore a bright fringe of order 2 is formed at M. **(1pt)**

b) $\frac{ax}{D} = 2 \lambda \Rightarrow x = \frac{2\lambda D}{a} = 2 i \quad (\frac{1}{2} \text{ pt})$

II – 1) $t = \frac{\ell}{v} \quad (\frac{1}{2} \text{ pt})$

2) $d = ct = \frac{c\ell}{v} = n\ell \quad (\frac{1}{2} \text{ pt})$

3) $n\ell - \ell = \ell(n-1) \quad (\frac{1}{2} \text{ pt})$

4) a) $\delta' = 0 \Rightarrow \frac{ax}{D} = (n-1)\ell$ et $x = 2 i \Rightarrow n = \frac{2\lambda}{\ell} + 1 \quad (\frac{1}{2} \text{ pt})$

b) $n = 1.0000025 \quad (\frac{1}{2} \text{ pt})$

5) a) $1.0000025 = 1 + 1.55 \times 10^{-6} y \Rightarrow y = 1.61. \quad (\frac{1}{2} \text{ pt})$

b) $y = 1.61 > 0.5$, therefore the air of the room is harmful. $(\frac{1}{4} \text{ pt})$

Fourth exercise : (7 ½ pts)

I - 1) $M.E = K.E + P.E_e + P.E_g = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 + 0 \quad (\frac{1}{2} \text{ pt})$

2) $M.E = cte \Rightarrow \frac{dM.E}{dt} = 0 = mvv' + kxv \Rightarrow x'' + \frac{k}{m}x = 0 \quad (\frac{1}{2} \text{ pt})$

3) $\omega = \sqrt{\frac{k}{m}} = 10 \text{ rad/s}$; $v = -X_m \omega \sin(\omega t + \varphi)$, for $t = 0$, $v = -X_m \omega \sin \varphi = 0$ and $x_0 = X_m \cos \varphi = d > 0 \Rightarrow \sin \varphi = 0 \Rightarrow \varphi = 0$ and $X_m = 5 \text{ cm} \quad (1 \frac{1}{4} \text{ pt})$

II - 1) During motion the magnetic flux that traverses the circuit AMNC is:

$\phi = BS \cos \theta$; S varies \Rightarrow flux varies \Rightarrow the induced e.m.f $e = - \frac{d\phi}{dt}$ exists **(1 pt)**

2) a) $\phi = BS \cos \theta = B(L_0 + x) \ell \quad (\frac{1}{2} \text{ pt})$

b) $e = - \frac{d\phi}{dt} = - B \ell \frac{dx}{dt} = - B \ell v \quad (\frac{1}{2} \text{ pt})$

c) The induced current is given by $i = \frac{e}{R} = \frac{-B\ell v}{R} \quad (\frac{1}{2} \text{ pt})$

d) $v > 0 \Rightarrow i < 0 \Rightarrow$ The induced current circulates in the negatives direction of the circuit (from M to N in the rod). $(\frac{1}{2} \text{ pt})$

3) a) During motion, the rod that is within the magnetic field is submitted to Laplace force \vec{F} , according to Lenz law it should opposes the causes that producing it, therefore \vec{F} opposes the displacement of the rod and plays the role of the force of friction that causes the damping of the oscillation and finally stops the rod. Therefore the motion is damped. **(1 pt)**

b) At $t_0 = 0$, $M.E = \frac{1}{2} k x_m^2 = 0.0625 \text{ J.} \quad (\frac{1}{2} \text{ pt})$

c) $|\Delta M.E| = |0 - 0.0625| = 0.0625 \text{ J.} \quad (\frac{1}{2} \text{ pt})$

d) In the form of electrical energy (or thermal in R). $(\frac{1}{4} \text{ pt})$