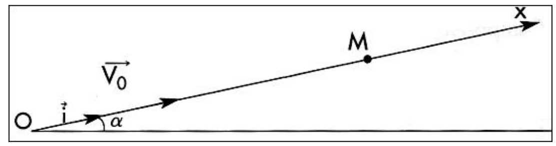
دورة سنة 2003 الاستثنائية	امتحانات شهادة الثانوية العامة فرع العلوم العامة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم : الرقم :	مسابقة في الفيزياء المدة : ثلاث ساعات	

This exam is formed of four obligatory exercises in four pages
The use of non-programmable calculators is allowed

<u>First Exercise</u> (6 points) Graphical study of energy exchange



Consider an inclined plane that makes an angle α with the horizontal (sin α = 0.2) and a marble (B) of mass m = 100 g, taken as a particle. We intend to study the energy exchange between the system (marble, Earth) and the surroundings.

To do that, the marble (B) is given, at the instant $t_o = 0$, the velocity $\vec{V}_o = V_0\,\vec{i}$ along the line of greatest slope OX. Given $V_o = 4\ m.s^{-1}$ and $g = 10\ m/s^2$.

The horizontal plane through point O is taken as the gravitational potential energy reference.

A- The forces of friction are supposed negligible.

1- Determine the value of the mechanical energy M.E of the system (marble, Earth).

2- At the instant t, the marble passes through a point M of abscissa OM = x. Determine, as a function of x, the expression of the gravitational potential energy $P.E_g$ of the system (marble, Earth) when the marble passes through M.

3-a) Trace, on the same system of axes, the curves representing the variations of the energies M.E and $P.E_g$ as a function of x.

Scale: - on the axis of abscissas: 1 cm represents 1 m;
- on the axis of energy: 1 cm represents 0.2 J.

b) Determine, using the graph, the speed of the marble for x = 3 m.

c) Determine, using the graph, the value of x_m of x for which the speed of (B) is zero.

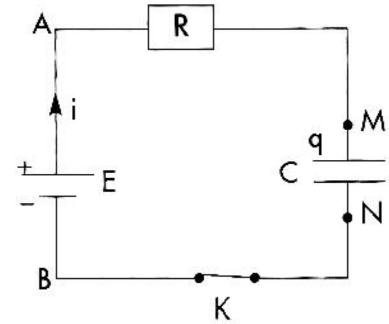
B-1. In reality, the speed of the marble becomes zero at a point of abscissa x = 3 m. The forces of friction are no longer negligible. Calculate then the work done by the forces of friction between x = 0 and x = 3 m.

2. The system (marble, Earth) thus exchanges energy with its surroundings. In what form and by how much?

<u>Second Exercise</u> (7 points) Response of an RC series circuit

The object of this exercise is to distinguish the response of an RC series circuit when we apply across its terminals a constant voltage, from its response when it carries a constant current.

A- Case of a constant voltage



The circuit of the adjacent figure allows us to charge the capacitor of capacitance C = 10 μF through a resistor of resistance R = 100 kΩ, under a constant voltage E = 9V. Take the instant t = 0 the instant when the switch K is closed.

1- Denote by $u_C = u_{MN}$, the instantaneous value of the voltage across the terminals of the capacitor.

a- Show that the differential equation in u_C is of the form:

$$u_C + RC\frac{du_C}{dt} = E$$

b- Knowing that the solution of this equation has the form: $u_C = A(1 - e^{-\frac{t}{\tau}})$ determine A and τ.

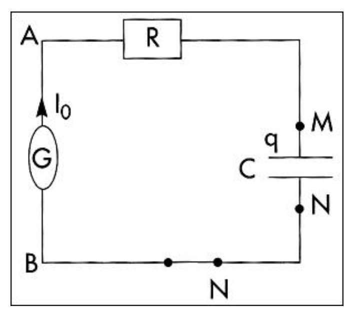
c-Trace the shape of the curve that gives the variation of u_C as a function of time.

2- a- Determine the expression of the voltage $u_R = u_{AM}$ as a function of time.

b- Trace, on the same system of axes, the shape of the curve giving the variation of u_R as a function of time.

3- What is the value of the interval if time t_A at the end of which u_C becomes practically 9V?

B- Case of a constant current



The preceding capacitor being discharged is to be recharged through the same resistor by a generator giving a constant current $I_o = 0.1$ mA.

1-a- Show that the charge q can be written, in SI, in the form $q = 10^4 \times t$.

b- The voltage $u_R = u_{AM}$ across the resistor remains constant. Determine its value.

c- Trace the shape of the graph representing u_R.

2-a- Determine the expression of the voltage $u_C = u_{MN}$ as a function of time.

b- Trace the shape of the graph representing u_C.

c- Determine the time interval t_B needed for the voltage u_C to attain the value 9 V.

C- Conclusions

1- Using the preceding graphs, specify the case where the voltage across the capacitor attains, in the steady state, a limiting value.

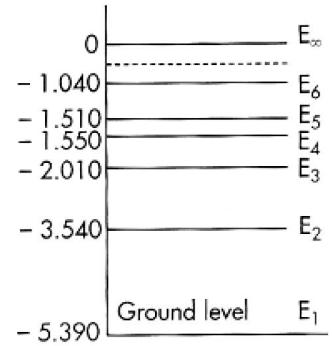
2- A camera is equipped with a flash that is formed of the preceding RC circuit. We intend to take the largest number of photos in a given time interval. To do so we have to charge the capacitor. Which one of the two preceding charging modes is more convenient? Why?

<u>Third exercise</u> (6 ½ points) The isotope $^{7}_{3}Li$ of lithium

As all the other chemical elements, the isotope $^{7}_{3}Li$ has properties that distinguish it from other chemical elements.

The object of this exercise is to show evidence of some properties of the isotope $^{7}_{3}Li$.

A- Emission spectrum of the lithium atom



The adjacent figure represents the energy levels of the lithium atom.

1-Calculate, in joule, the energy (E_l) of the atom when it is in the ground state and (E_5) when it is in the fifth state.

2-During the downward transition (de-excitation) from different energy levels to the ground level, the lithium atom emits some radiations.

a- Calculate the highest and the lowest frequency of the emitted radiations.

b- The corresponding emission spectrum is discontinuous. Why?

3- The lithium atom, being in the ground state, captures:

a- a photon whose associated radiation has a wavelength of $\lambda = 319.9$ nm. Show that the atom absorbs this photon. In what level would it be?

b- a photon of energy 6.02 eV. An electron is thus liberated. Calculate, in eV, the kinetic energy of that electron.

B- Nuclear reaction

A nucleus $^{A}_{Z}X$, at rest, is bombarded by a proton carrying an energy of 0.65 MeV; we obtain two α particles.

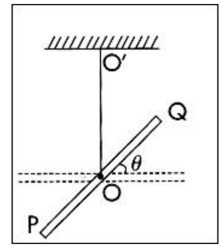
1- Is this nuclear reaction spontaneous or provoked? Justify your answer.
2- Determine the values of Z and A by applying the convenient conservation laws. Identify the nucleus X.
3- Calculate the mass defect due to this reaction and deduce the corresponding energy liberated.
4- Knowing that the two obtained α particles have the same kinetic energy E. Calculate E.

Given: $h = 6.62 \times 10^{-34}$ J x s; $c = 3 \times 10^{8}$ m.s^{-1}; 1 eV = 1.6×10^{-19} J;
1 u = 931.5 MeV/c^{2};
mass of the nucleus of lithium: m (Li) = 7.01435 u;
mass of the a particle: m(α) = 4.00150 u;
mass of a proton: m_p = 1.00727 u.

Fourth exercise (7½points) Moment of inertia of a rigid rod

The object of this exercise is to determine, by two methods, the moment of inertia I_o of a rigid and homogeneous rod PQ of negligible cross-section, about an axis perpendicular to it through its mid-point O. In order to do that we consider the rod PQ of mass M = 375 g and of length l = 20 cm. We neglect all frictions. Take $g = 10\ m/s^2$ and $\pi^2 = 10$.

A- Case of a torsion pendulum

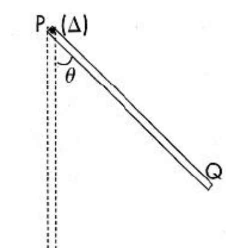


The rod PQ, being horizontal, its mid point O is fixed to a vertical torsion wire OO' whose torsion constant is $C = 5 \times 10^{-4}$ SI; the other end O' of the wire is fixed to a support. We thus obtain a torsion pendulum. The rod PQ, in the horizontal plane, is shifted from its equilibrium position around the vertical axis OO' by $\theta_m = 0.1$ rad in a direction taken as positive and is released from rest at the instant $t_o = 0$.
PQ thus oscillates around OO' in the horizontal plane around its equilibrium position.
At any instant t during its motion, the position of the rod is defined by its angular elongation θ with its equilibrium position.

1- a- Write, at the instant t, the expression of the mechanical energy M.E of the pendulum as a function of I_o, C, θ and the angular speed θ'.
b- Calculate the value of M.E.
c- Derive the second order differential equation that describes the motion of the pendulum.
d- Prove that the expression of the proper period T_o can be written as $T_o = 2\pi\sqrt{\frac{I_o}{C}}$.

2- We measure the time t_1 for 10 oscillations; we find $t_1 = 100$ s. Calculate I_o.

B- Case of a compound pendulum



The rod PQ, alone, is now free to rotate in the vertical plane around a horizontal axis (Δ) passing through its extremity P.
The rod PQ is shifted by an angle $\theta_m = 0.1$ rad from its equilibrium position and is then released from rest at the instant $t_o = 0$. The rod PQ thus oscillates around its equilibrium position.
At any instant t, the position of the rod is defined by the angular elongation θ with its equilibrium position.
The horizontal plane containing (Δ) is taken as the gravitational potential energy reference.

1-a- Write the expression of the mechanical energy M.E of the system (rod, Earth) at any instant t as a function of M, g, l, θ, the angular speed and the moment of inertia I_1 of the rod about the axis (Δ).
b- Calculate the value of M.E.
c- Derive the second order differential equation that describes the motion of the rod.
d- Prove that the expression for the proper period T_o' may be written as: $T_o' = 2\pi\sqrt{\frac{2I_1}{Mgl}}$

2- We measure the time t2 of 10 oscillations of the rod. We find $t_2 = 7.3$ s. Calculate I_1.

3- Knowing that $I_1 = I_o + \frac{Ml^2}{4}$, find again the value of I_o.

Take, for $\theta \le 0.2$ rad, $\sin\theta = \theta$ rad and $(1 - \cos\theta) = \frac{\theta^2}{2}$ (θ in rad).

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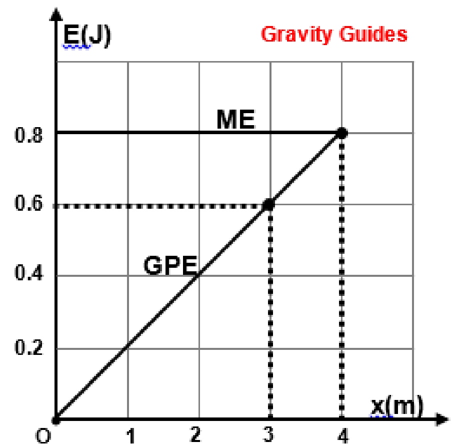
Question I (6.5 points)		
A-1.	$ME = ME_O = KE_O + GPE_O$. Thus $ME = 0.8J + 0 = 0.8J$	1
A-2.	$GPE = mgx \sin\alpha$ $GPE = 0.1 \times 10 \times x \times 0.2 = 0.2x$ (x in m, and GPE in J).	0.5
A-3.a)		0.5 0.5 0.25 0.25
A-3.b)	For $x = 3m$, we find graphically $GPE = 0.6J$; So, $KE = \frac{1}{2}mv^2 = 0.2J$; thus, $v = 2m/s$	0.75
A-3.c)	$KE = 0$; so $ME = GPE$, Graphically the abscissa is $x_m = 4m$.	0.75
B-1.	$ME_2 = KE_2 + GPE_2 = 0 + mgx \sin\alpha = 0.6J$. $W_{\vec{f}} = \Delta(ME) = ME_2 - ME_O = 0.6J - 0.8J = -0.2J$	1 0.5
B-2.	The energy exchanged with the surroundings is converted into thermal energy that appears in the form of heat.	0.5

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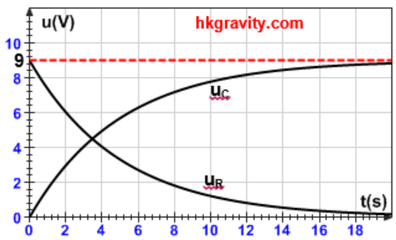
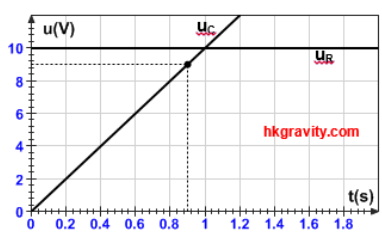
Question II (07 points)		
A-1.a)	Law of addition of voltages $u_{AB} = u_{AM} + u_{MN}$, so $E = u_C + u_R$; Thus, $E = u_C + RC\frac{du_C}{dt}$.	**0.5**
A-1.b)	We have $u_C = A\left(1 - e^{-\frac{t}{\tau}}\right)$, so $\frac{du_C}{dt} = \frac{A}{\tau}e^{-\frac{t}{\tau}}$; $E = A + e^{-\frac{t}{\tau}}\left(1 - \frac{RC}{\tau}\right)$. This equation is verified at any instant t, and $A\,e^{-\frac{t}{\tau}} \neq 0$, then $A = E$ and $\tau = RC$.	**1**
A-1.c)		**0.5** **0.5**
A-2.a)	$u_R = RC\frac{du_C}{dt} = E\,e^{-\frac{t}{\tau}}$.	**0.5**
A-2.b)	Graph of $u_R(t)$.	
A-2.c)	The capacitor becomes practically charged after a duration: $t_A = 5\,\tau = 5\,RC = 5 \times (100 \times 10^3) \times (10 \times 10^{-6}) = 5\,s$	**0.5**
B-1.a)	$q = I_0 t + q_0$ & $q_0 = 0$; Thus, $q = 10^{-4}\,t$ where t in s & q in C.	**0.5**
B-1.b)	The voltage across the resistor is given by: $u_R = R\,i = R\,I_0 = 10\,V$	**0.5**
B-1.c)	Graph 	**0.25** **0.5**
B-2.a)	Thus, $u_C = \frac{10^{-4}}{10^{-5}}t = 10\,t$ (where t in s & u_C in V)	**0.5**
B-2.b)	The curve representing u_C is a straight line passing through origin.	
	$q = 10^{-4}\,t$, thus $t_B = \frac{9 \times 10^{-5}}{10^{-4}} = 0.9\,s$	**0.5**
C-1.	Mode A	**0.25**
C-2.	Mode B, charging faster and more photos are possible	**0.5**

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Question III (6.5 points)		
A-1.	$E_1 = -5.39\ eV = -5.39 \times 1.6 \times 10^{-19} J = -8.624 \times 10^{-19} J$; and $E_6 = -1.040\ eV = -1.040 \times 1.6 \times 10^{-19} J = -1.664 \times 10^{-19} J$	**0.25** **0.25**
A-2.a)	To the ground state $n = 1$ from the nearest energy level $n = 2$: $h\,\nu_{\min} = E_2 - E_1$, then $\nu_{\min} = 4.5 \times 10^{14} Hz$ The farthest energy levels, from $n = \infty$ to the ground state: $h\,\nu_{\max} = E_\infty - E_1$, then $\nu_{\max} = 1.30 \times 10^{15} Hz$.	**0.75** **0.75**
A-2.b)	The energy levels are quantified,	**0.5**
A-3.a)	$E_{ph} = \frac{hc}{\lambda} = 6.208 \times 10^{-19} J = 3.880\ eV$ $E_{ph} + E_1 = E_5$, to the 5th energy level.	**0.5** **0.25**
A-3.b)	The ionization energy: $W_0 = E_\infty - E_1 = 5.390\ eV$; $E_{ph} > W_0$; $KE = E_{Ph} - W_0 = 6.020eV - 5.390eV = 0.630eV$	**0.25** **0.5**
B-1.	Intervention of a proton (external agent).	**0.5**
B-2.	Conservation of mass number: $A = 7$; Conservation of charge number: $Z = 3$; The nucleus is the lithium ${}^{7}_{3}Li$.	**0.25** **0.25** **0.25**
B-3.	$m_\ell = m({}^{7}_{3}Li) + m({}^{1}_{1}H) - 2\,m({}^{4}_{2}He)$; $m_\ell = 0.01862u$; The energy liberated is $E_\ell = m_\ell\, c^2 = 0.01862 \times 931.5 MeV = 17.34\ MeV$.	**0.5** **0.25**
B-4.	Conservation of energy: $E_\ell = m_\ell c^2 = 2KE_\alpha - \left(KE_p + KE_{Li}\right)$; Then, $KE_\alpha = \frac{17.34MeV + 0.65MeV}{2} \approx 9MeV$	**0.5**

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Question IV (7.5 points)		
A-1.a)	$ME = KE + PE_e = \frac{1}{2} I_0 \theta'^2 + \frac{1}{2} C\theta^2$	**0.25**
A-1.b)	The mechanical energy is conserved: $$ME = \frac{1}{2} C\theta_m^2 = 2.5 \times 10^{-6} J.$$	**0.75**
A-1.c)	The mechanical energy of the pendulum is conserved, then: $\frac{d(ME)}{dt} = 0, I_0\theta'\theta'' + C\theta\theta' = 0$; (however $\theta' \neq 0$, in motion) Thus, $\theta'' + \frac{C}{I_0}\theta = 0$	**1**
A-1.d)	The differential equation is of second order where $\omega_0^2 = \frac{C}{I_0}$; Then $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I_0}{C}}$.	**0.25** **0.25**
A-2.	$T_0 = \frac{100}{10} = 10s$; Then $I_0 = 1.25 \times 10^{-3} kg.m^2$.	**0.25** **0.5**
B-1.a)	$ME = KE + GPE = \frac{1}{2} I_1 \theta'^2 - M\,g\frac{\ell}{2}\cos\theta$.	**0.25**
B-1.b)	$ME = -\frac{1}{2} M\,g\ell\cos\theta_m = 0.373J$	**0.75**
B-1.c)	$\frac{d(ME)}{dt} = 0$, thus, $\theta'' + \frac{Mg\ell}{2I_1}\theta = 0$.	**1**
B-1.d)	$\omega_0'^2 = \frac{Mg\ell}{2I_1}$; Then proper period: $T_0' = \frac{2\pi}{\omega_0'} = 2\pi\sqrt{\frac{2I_1}{M\,g\ell}}$	**0.25** **0.5**
B-2.	$I_1 = \frac{Mg\ell\, T'^2}{8\,\pi^2} = \frac{0.375\times10\times0.2\times0.73^2}{8\times10} = 5\times10^{-3} kg.m^2$	**0.5**
B-3.	$I_0 = I_1 - \frac{M\ell^2}{4} = 5\times10^{-3} - \frac{0.375\times0.2^2}{4} = 1.25\times10^{-3} kg.m^2$	**1**