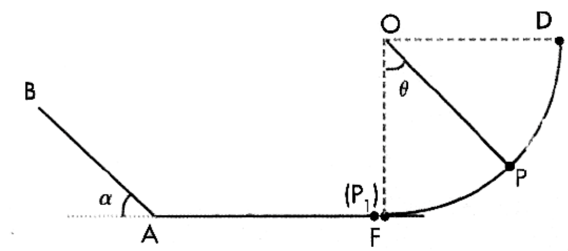
وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات شهادة الثانوية العامة *فرع العلوم العامة*	دورة سنة 2002 العادية
	مسابقة في الفيزياء المدة : ثلاث ساعات	الاسم : الرقم :

This exam is formed of four obligatory exercises in four pages
The use of non-programmable calculators is allowed

<u>First Exercise</u> (7 points) Conservation and non-conservation of the mechanical energy

Consider a material system (S) formed of an inextensible and mass less string of length l = 0.45 m, having one of its ends O fixed while the other end carries a particle (P) of mass m = 0.1 kg.
Take $g = 10\ m/s^2$.



1) (S) is shifted from its equilibrium position by $\theta_m = 90°$, while the string is under tension, and then released without initial velocity.
Take the horizontal plane containing FA as a gravitational potential energy reference for the system [(S), Earth]. We neglect friction on the axis through O and air resistance.

a) Calculate the initial mechanical energy of the system [(S), Earth] when (P) was at D.
b) Determine the expression of the mechanical energy of the system [(S), Earth] in terms of l, m, g, V and θ, where V is the speed of (P) when the string passes through a position making an angle θ with the vertical.
c) Determine the value of θ, (0° < θ < 90°), for which the kinetic energy of (P) is equal to the gravitational potential energy of the system [(S), Earth].
d) Calculate the magnitude V_o of the velocity $\vec{V}_o$ of (P) as it passes through its equilibrium position.

2) Upon passing through the equilibrium position, the string is cut, and (P) enters in a head-on collision with a stationary particle (P_1) of mass ml= 0.2 kg. As a result, (P_1) is projected with a velocity $\vec{V}_1$of magnitude V_1 = 2m/s. Determine the magnitude V of the velocity $\vec{V}$ of (P) right after impact knowing that $\vec{V}_o$, $\vec{V}_1$, and $\vec{V}$ are collinear.
Is the collision elastic? Justify your answer.

3) (P_1), being projected with a speed V_1 = 2m/s, moves along the frictionless horizontal track FA, and rises at A with the speed V_1, along the line of greatest slope of the inclined plane AB that makes an angle α = 30° with the horizontal.

a) Suppose now that the friction along AB is negligible. Determine the position of the point M at which (P_1) turns back.

b) In fact, AB is not frictionless; (P_1) reaches a point N and turns back, where AN = 20 cm. Calculate the variation in the mechanical energy of the system [(P_1), Earth] between A and N, and then deduce the magnitude of the force of friction (assumed constant) along AN.

<u>Second Exercise</u> (6 ½ points) **Determination of the capacitance of a capacitor**

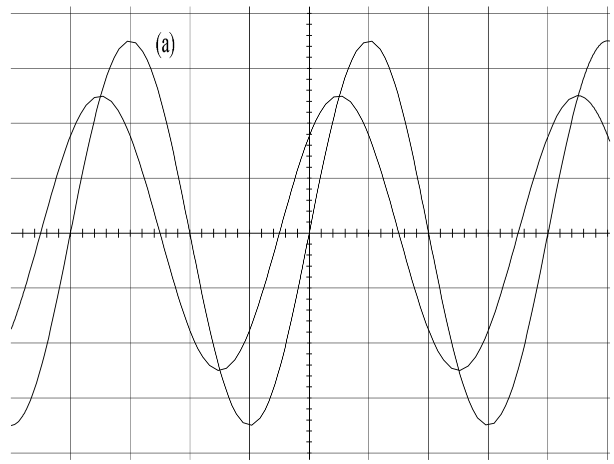
In order to determine the capacitance C of a capacitor, we use the following components:

- a function generator (LFG) delivering an alternating sinusoidal voltage: $v = V_m \cos \omega t$ (v in V and tin s), a resistor of resistance $R = 50\ \Omega$, a coil of inductance $L = 0.16$ H and of negligible resistance, an oscilloscope and connecting wires. Take $0.32\ \pi = 1$.

A) In a first experiment, we connect the capacitor in series with the resistor across the LFG. The oscilloscope is used to display the voltage v across the LFG on the channel Yl and the voltage vR across the resistor on the channel Y2. The adjustments of the oscilloscope are:

vertical sensitivity: 2 V/division on both channels,

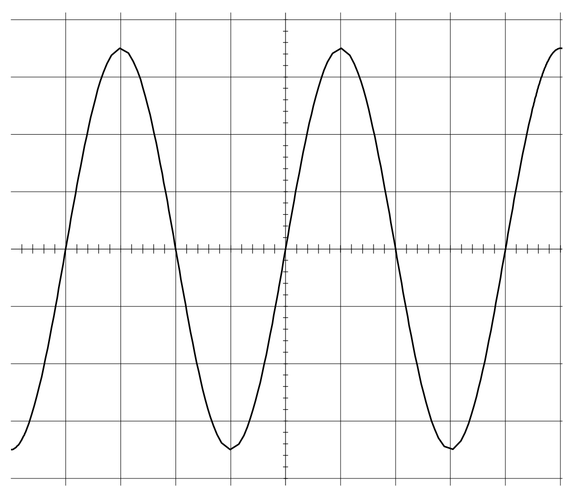
horizontal sensitivity: 5 ms/division.



1) Draw again a diagram of the circuit showing on it the connections of the oscilloscope.
2) The waveforms displayed are represented as in the adjacent figure:
   a) Waveform (a) represents v. Why?
   b) Determine the frequency of the voltage v and the phase difference between v and vR.
   c) Using the numerical values of Vm and w, write the expressions of v and of vR as a function of time and deduce the expression of the instantaneous current i in the circuit.
   d) Knowing that the voltage vC across the capacitor is $v_C = \frac{q}{C}$ show that uC is given by

$$v_C = \frac{3.2\text{x}10^{-4}}{C} \cos(\omega t - \frac{\pi}{4})$$

   e) Determine the value of C using the law of addition of voltages by taking a particular value of the time t.



B) In a second experiment, we insert the coil in series with the previous circuit.
We thus obtain an RLC series circuit and we keep the same connections of the oscilloscope.
We observe only one waveform on the screen (the two waveforms are confounded).
The above result shows evidence of an electric phenomenon that took place.
Name this phenomenon and calculate again the value of the capacitance C.

Third exercise (6.5 points) Radioactivity

Given the masses of the nuclei: m($^{131}_{53}I$) = 130.87697 u; m($^{A}_{Z}Xe$) = 130.87538 u; mass of an electron = 5,5 x 10^{-4} u;

1 u = 931.5 MeV/c^2; 1 MeV = 1.6 x 10^{-13} J; h = 6,63 x 10^{-34} J.s et c = 3x10^8m/s

In order to detect a trouble in the functioning of the thyroid, we inject it with a sample of an iodine radionuclide $^{131}_{53}I$. This radionuclide has a period (half-life) of 8 days and it is a β^- emitter. The disintegration of the nuclide $^{131}_{53}I$ gives rise to a daughter nucleus $^{A}_{Z}Xe$ supposed at rest.

1) a) The disintegration of a nucleus of $^{131}_{53}I$ is accompanied by the emission of a γ radiation. Due to what is this emission?

b) Write the equation of the disintegration of $^{131}_{53}I$ nucleus.

c) Calculate the decay constant of the radionuclide. Deduce the number of the nuclei of the sample at the instant of injection, knowing that the activity of the sample, at that instant, is 1.5 x 10^5 Bq.

d) Calculate the number of the disintegrated nuclei at the end of 24 days.

2) a) Calculate the energy liberated by the disintegration of one nucleus of $^{131}_{53}I$.

b) Calculate the energy of a γ photon knowing that the associated wavelength is 3.55 x 10^{-12}m.

c) The energy of an antineutrino being 0.07 MeV, calculate the average kinetic energy of an emitted electron.

d) During the disintegration of the $^{131}_{53}I$ nuclei, the thyroid, of mass 40 g, absorbs only the average kinetic energy of the emitted electrons and that of γ photons. Knowing that the dose absorbed by a body is the energy absorbed by a unit mass of this body, calculate, in J/Kg, the absorbed dose by that thyroid during 24 days.

Fourth exercise (7½points) Effect of the resistance of a resistor in an electric circuit

According to the value of the resistance of the resistor in a circuit, the steady state is attained slower or faster, or the circuit may be (or may not be) the seat of ideal electric oscillations.
In this exercise, we intend to show the effect of the resistance of a resistor in some electric circuits.
Given a resistor (R) of adjustable resistance R, a coil (B) of inductance L = 0.64 H and of negligible resistance, a capacitor (C) of capacitance $C = 10^{-6}$ F, a generator (G) of negligible internal resistance and of electromotive force E, a switch (K), an oscilloscope with a memory and connecting wires.

A) Case of an R-L series circuit

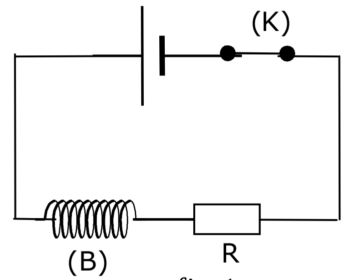


fig 1

We connect up the (R- L) series circuit of figure 1. The switch (K) is closed at t = 0.

1) Derive, in the transient state, the differential equation in $v_R = Ri$ associated with the considered circuit.
2) The expression $v_R = V_0(1 - e^{-\frac{t}{\tau}})$ is a solution of this differential equation. Deduce the expression of U_0 and τ in terms of E, R and L.
3) Express, in terms of τ, the time t at the end of which the steady state is practically attained. What is then the value of the voltage across the coil?
4) a) Compare the values of t and of V_o corresponding to:
   i) $R_1 = 12\ \Omega$; ii) $R_2 = 60\ \Omega$; iii) $R_3 = 600\ \Omega$.
   b) Draw, on the same system of axes (t, v_R), the shape of the curve that represents v_R for each value of R.
   c) What is then the role of the value of R in the growth of the current towards the steady state?

B) Case of an R-C series circuit

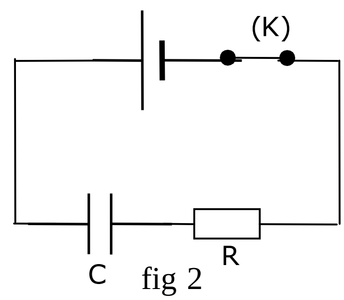


fig 2

We connect up the (R-C) series circuit of the figure 2. We close (K) at t = O.

1) Derive, in the transient state, the differential equation in $v_C = \frac{q}{C}$ associated with the considered circuit, q being the charge of the armature A of the capacitor.
2) The expression $v_C = V_C(1 - e^{-\frac{t}{\tau'}})$ is a solution of this differential equation. Deduce the expressions of V_C and of τ' in terms of E, R and C.
3) a) Express, in terms of τ', the time t' at the end of which the steady state is practically attained. What is then the value of the voltage across the terminals of the resistor?
   b) Compare the values of t' and V_C corresponding to:
   i) $R_1 = 12\ \Omega$; ii) $R_2 = 60\ \Omega$; iii) $R_3 = 600\ \Omega$.
   c) Draw, on the same system of axes (t, v_c), the shape of the curve representing u_c for each value of R.
   d) What is then the role of the value of R in the decay of the current towards the steady state?

C) Case of an RLC series circuit

(C), being charged, is connected with (B) and (R) thus forming an RLC series circuit. This circuit is the seat of free electric oscillations.

The oscilloscope, connected across the terminals of (C), would display the variation of v_C as a function of time.

If R takes the value R = 0 and the switch (K) is closed at t = 0, we observe on the screen of the oscilloscope the waveform of figure 3.

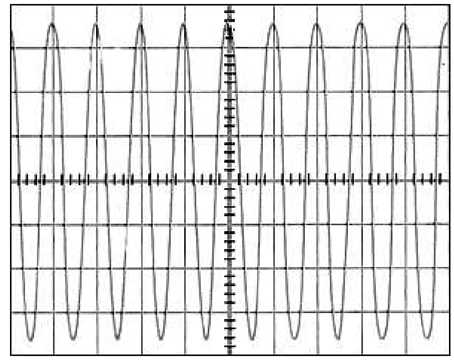


fig 3

If we give R a certain value and we close the switch at t = 0, we obtain the waveform of figure 4 without changing the adjustments of the oscilloscope.

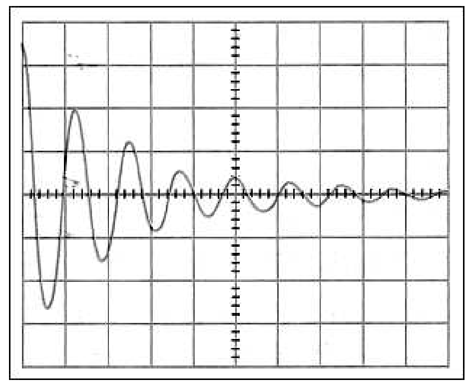


fig 4

1) Give the expression of the proper (natural) period T_0 of the RLC series circuit thus formed and calculate its value.

2) Determine, using the waveform of figure 3, the time base (horizontal sensitivity) used.

3) a) In which case are the oscillations undamped? Why?

b) Calculate the value of the pseudo-period T of oscillations.

4) Compare the values of T and T_0. Conclude.

Solution

First Exercise (7 points)

1)
a) At D: KE = 0 J car v = 0 m/s

$P.E_g = mgl = 0.1x10x0.45 = 0.45$ J

$M.E = KE + P.E_g = 0.45$ J (0.75 pt)

b)

$M.E = KE + P.E_g = \frac{1}{2}mv^2 + mgh;$ et h = l-lcosθ (0.75 pt)

$M.E = \frac{1}{2}mv^2 + mgl(1-\cos\theta)$

c) M.E of the system [(S), Terre] is conserved because friction is neglected.

$M.E = M.E_D = 0.45$ J.

$P.E_g = K.E = \frac{M.E}{2} = 0.45 \text{ J} \Rightarrow P.E_g = mgl(1-\cos\theta) = 0.45 \Rightarrow \theta = 60^o$ (1 pt)

d) $M.E = M.E_F = 0.45$ J ; $P.E_{gF} = 0$.

$K.E = \frac{1}{2}mV_o^2 = 0.45 \Rightarrow V_o = 3$ m/s (0.5 pt)

2) During collision, the linear momentum of the system (P, P_1) is conserved:

$m\vec{V}_o = m\vec{V} + m_1\vec{V}_1$

$\vec{V}_o$, $\vec{V}$ et $\vec{V}_1$ are collinear: $mV_o = mV + m_1V_1 \Rightarrow V = \frac{mV_o - m_1V_1}{m_1} = -1$ m/s (1 pt)

$K.E_i$ of the system before collision: $K.E_i = \frac{1}{2}mV_o^2 = 0.45$ J.

$K.E_f$ of the system before collision: $K.E_f = \frac{1}{2}mV^2 + m_1V_1^2 = 0.45$ J.

$K.E_i = K.E_f \Rightarrow$ the collision is elastic. (0.75 pt)

3)
a) At A, $P.E_{gA} = 0$ J $\Rightarrow M.E_A = K.E_A = \frac{1}{2}m_1V_A^2 = 0.4$ J.

M.E of the system [(S), Terre] is conserved because friction is neglected, $M.E_A = M.E_M$

At M, $E_{cM} = 0$ J $\Rightarrow E_{mM} = E_{pM} = m_1gAM\sin\alpha = 0.4 \Rightarrow AM = 0.4$ m. (1 pt)

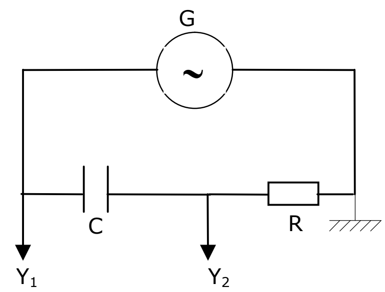
b) At N, $K.E_c = 0$ J $\Rightarrow M.E_N = P.E_{gN} = m_1gAN\sin\alpha = 0.2$ J

$\Delta E_m = E_{mN} - E_{mA} = -0.20$ J

$\Delta E_m = W_{\vec{f}} = \vec{f}.\overrightarrow{AN} = -f \times AN \Rightarrow f = \frac{-\Delta E_m}{AN} = \frac{0.2}{0.2} = 1$ N (1.25 pts)

Second Exercise **(6 ½ points)**

1) (0.5 pt)



2)

a) $V_{m(a)} > V_{mb}$ (0.5 pt)

b) $T = 4(\text{div}) \times 5 = 20$ ms $\Rightarrow f = \frac{1}{T} = 50$ Hz

$T \rightarrow 4 \text{ div} \rightarrow 2\pi$

$0{,}5 \text{ div} \rightarrow \varphi \quad \Rightarrow \varphi = \frac{\pi}{4}$

v is lags behind i or v_R by $\frac{\pi}{4}$rad. (1.25 pts)

c) $\omega = 2\pi f = 100\pi$ rad/s

$v = 7\cos 100\pi t$.

$V_{Rm} = 2{,}5(\text{div}) \times 2 = 5$ V

$v_R = 5\cos(100\pi t + \frac{\pi}{4})$ and $i = \frac{v_R}{R} = 0{,}1\cos(100\pi t + \frac{\pi}{4})$ (1.75 pts)

d) $i = \frac{dq}{dt} \Rightarrow q = \int i\, dt \Rightarrow u_C = \frac{q}{C} = \frac{1}{C}\int i\, dt = \frac{1}{C}\int [0{,}1\cos(100\pi t + \frac{\pi}{4})]\, dt = \frac{3{,}2 \times 10^{-4}}{C}\cos(\omega t - \frac{\pi}{4})$ (0.5 pt)

e) $v_G = v_R + v_C = Ri + v_C$

$7\cos 100\pi t = 5\cos(100\pi t + \frac{\pi}{4}) + \frac{3{,}2 \times 10^{-4}}{C}\cos(\omega t - \frac{\pi}{4})$

for t = 0: $7 = 5\frac{\sqrt{2}}{2} + \frac{3{,}2 \times 10^{-4}}{C} \times \frac{\sqrt{2}}{2} \Rightarrow C = 64 \times 10^{-6}\text{ F} = 64\ \mu\text{F}$. (1 pt)

B- The phenomenon is the current resonance.

$f = f_o = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{4\pi^2 f_o^2 L} = 64 \times 10^{-6}\text{ F} = 64\ \mu\text{F}$ (1 pt)

Third exercise (6.5 points)

1)

a) The emission of γ ray is due to the de-excitation of the daughter nucleus. (0.25 pt)

b) $^{131}_{53}I \rightarrow {}^{A}_{Z}Xe + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + {}^{0}_{0}\gamma$

The law of conservation of charge number gives: 53 = Z – 1 thus Z = 54.

The law of conservation of mass number gives: 131 = A thus A = 131. (0.75 pt)

c) $\lambda = \frac{\ln 2}{T} = \frac{0.693}{T_{(s)}} = 10^{-6}$ s. (0.5 pt)

$A_o = \lambda N_o \Rightarrow N_o = \frac{A_o}{\lambda} = 1.5 \times 10^{11}$ nuclei (0.5 pt)

d) t = 24 days = 3 T, and the number of disintegrated at the end of 3T is: $N\text{-}N_o$

$N = \frac{N_o}{2^3} \Rightarrow N - N_o = 1.31 \times 10^{11}$ nuclei (1 pt)

2)

a) (1 pt)

$E = \Delta m \times c^2 = (m_{before} - m_{after})c^2 = (0.00104) \times 931.5 = 0.96876$ MeV$= 0.96876 \times 1.6 \times 10^{-13} = 1.55 \times 10^{-13}$ J

b) $E_{ph} = \frac{hc}{\lambda} = 5.6 \times 10^{-14}$ J = 0.35 MeV (0.75 pt)

c) The principe of conservation of energy gives:

$E = K.E(Xe) + E_{ph} + E(\bar{\nu}) + K.E(\beta^-)$

$0.96876 = 0 + 0.35 + 0.07 + K.E(\beta^-) \Rightarrow K.E(\beta^-) = 0.55$ MeV $= 0{,}88 \times 10^{-13}$ J (0.5 pts)

d) The energy absorbed by the thyroid during the disintegration of a single nucleus is:

$E_1 = 0.55 + 0.35 = 0.9$ MeV

For t = 24 days, $E_2 = E_1 \times 1.31 \times 10^{11} = 1.18 \times 10^{11}$ MeV $= 1.89 \times 10^{-2}$ J

$D = \frac{E_2}{mass} = \frac{1.89 \times 10^{-2}}{0.04} = 0.47$ J/kg (1.25 pts)

Fourth exercise **(7½points)**

1) $v_G = v_R + v_B$; $v_R = Ri \Rightarrow i = \frac{v_R}{R}$; $v_B = L\frac{di}{dt} = \frac{L}{R}\frac{dv_R}{dt}$

$E = v_R + \frac{L}{R}\frac{dv_R}{dt}$ (0.5 pt)

2)

$v_R = V_o(1-e^{-\frac{t}{\tau}})$; $\frac{dv_R}{dt} = \frac{V_o}{\tau}e^{-\frac{t}{\tau}} \Rightarrow E = V_o(1-e^{-\frac{t}{\tau}}) + \frac{L}{R}x(\frac{V_o}{\tau}e^{-\frac{t}{\tau}})$

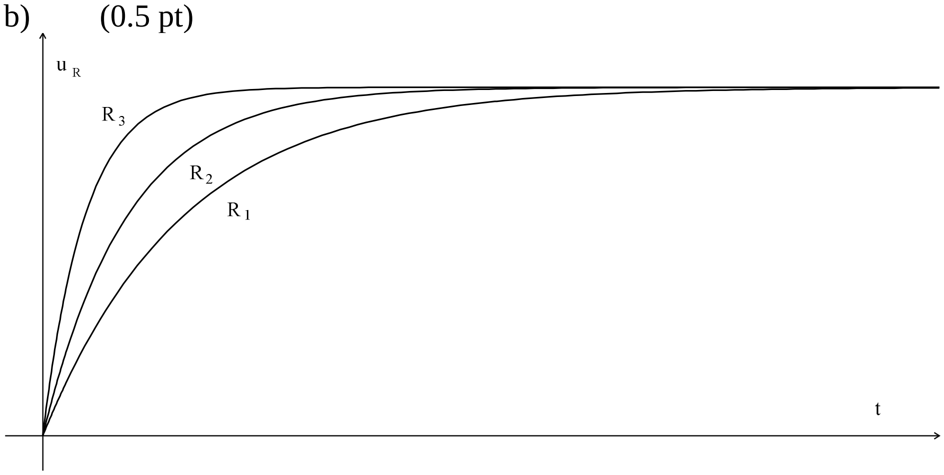
$E = V_o \times e^{-\frac{t}{\tau}}(\frac{L}{\tau\times R} - 1) + V_o \quad \forall\, t \Rightarrow$ (0.5 pt)

$E = V_o$; $\frac{L}{\tau\times R} - 1 = 0 \Rightarrow \tau = \frac{L}{R}$

3) The steady state is practically attained at the end of : $t = 5\tau$; $v_R \approx V_o$ (0.5 pt)

4) a)
i) $R_1 = 12\ \Omega$: $t_1 = 5\tau_1 = 0{,}267$ s; ii) $R_2 = 60\ \Omega$: $t_2 = 5\tau_2 = 0{,}053$ s ; iii) $R_3 = 600\ \Omega$: $t_3 = 5\tau_3 = 0{,}0053$ s
$V_o = E \quad \forall\ \tau$ (0.5 pt)

b) (0.5 pt)



c) When the resistor R increases, the steady state is attained more quickly. (0.25 pt)

B-

1) $v_G = v_R + v_C$; $v_R = Ri \Rightarrow i = \frac{dq}{dt} = C\frac{dv_C}{dt}$;

$E = RC\frac{dv_C}{dt} + v_C$ (0.5 pt)

2)

$$v_C = V_C(1 - e^{-\frac{t}{\tau'}}); \ \frac{dv_C}{dt} = \frac{V_C}{\tau'} e^{-\frac{t}{\tau}} \Rightarrow E = V_C(1 - e^{-\frac{t}{\tau'}}) + RCx(\frac{V_C}{\tau} e^{-\frac{t}{\tau}})$$

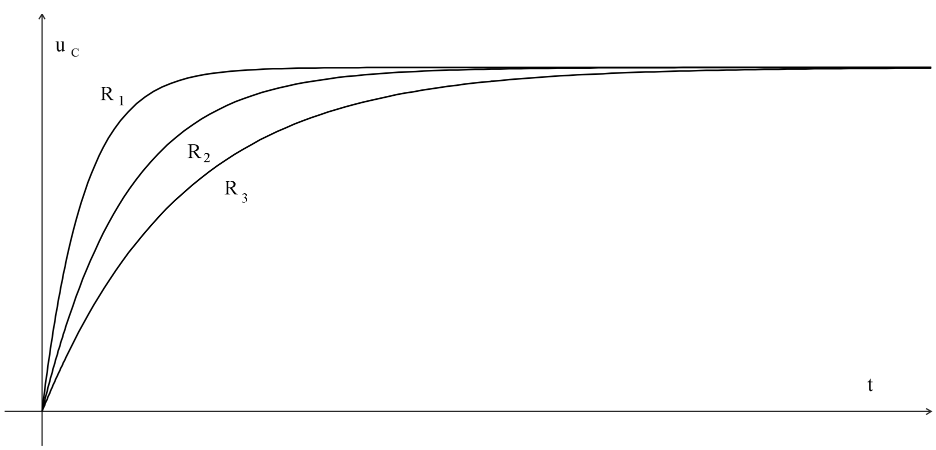
$$E = V_C \times e^{-\frac{t}{\tau'}}(\frac{RC}{\tau'} - 1) + V_C \quad \forall\ t \ \Rightarrow$$ (0.5 pt)

$$E = V_C; \ \frac{RC}{\tau'} - 1 = 0 \ \Rightarrow \tau' = RC$$

3)
a) The steady state is practically attained at the end of: t' = 5τ'; $U_C \approx E$ (0.5 pt)

b) i) $R_1 = 12\ \Omega$: $t'_1 = 5\tau'_1 = 6 \times 10^{-5}$ s; ii) $R_2 = 60\ \Omega$: $t'_2 = 5\tau'_2 = 30 \times 10^{-5}$ s ;
iii) $R_3 = 600\ \Omega$: $t'_3 = 5\tau'_3 = 6 \times 10^{-6}$ s
$U_C = E \ \ \forall\ \tau$ (0.5 pt)

c) (0.5 pt)



d) When the resistor R increases, the steady state is attained more slowly.

C-
1) $T_o = 2\pi\sqrt{LC} = 5 \times 10^{-3}$ s = 5 ms. (0.75 pt)
2) T_o = 1(div) x S_h then S_h = 5 ms/div (0.25 pt)

3)
a) The oscillations are damped because the amplitude decreases with time. (0.5 pt)

b) T = 1,25(div) x 5 = 6,25 ms (0.25 pt)

4) T > T_o: the pseudo-period increases with the resistance of the circuit. (0.25 pt)